CORRELATING GEL RHEOLOGY WITH
BEHAVIOR DURING EXTRUSION
THROUGH FRACTURES

By
Ying Wang

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New Mexico Institute of Mining and Technology
Socorro, New Mexico 87801
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ABSTRACT

In many successful conformance control treatments, large volumes of gels must be extruded through fractures during placement. The pressure gradient for gel extrusion depends strongly on fracture width and gel composition. Experiments that directly measured gel properties in fractures are both expensive and time-consuming. Therefore, we investigated whether rheology measurements can more cost-effectively assess gel properties in fractures.

Rheology behavior strongly paralleled our extrusion results. However, for a given aperture (fracture width or plate-plate separation), pressure gradients during gel extrusion were much greater than values expected from rheology measurements. Extensive experiments were performed to establish whether wall slip could be responsible for this discrepancy. Steady shear and oscillatory shear data were collected with a rheometer using smooth and rough parallel plate geometries, employing various gap heights. With smooth plates, wall-slip effects were noted. However, with rough parallel plates, wall slip was negligible and proven not to be responsible for the discrepancy between extrusion and rheological results. Due to the anisotropic microstructures or forces in the interior of the viscoelastic material, when gels were sheared in fractures first normal stress difference occurred and exerted extra pressure to the walls. This might result in extra pressure gradient along the fracture. Although first normal stress might result in extra pressure gradient on the wall due to narrower channels caused by “wormholes”, it was
not enough to explain the discrepancy. Instead, a logical explanation was that the aperture of the actual gel pathway is much smaller than the fracture width.

Considering the shear-thinning properties of the Cr(III)-acetate-HPAM gels, a model based on power-law equations was developed through shell momentum balance. The power-law model explained why the pressure gradient for gel extrusion varied inversely with the square of fracture width rather than inversely with fracture width. This model showed the pressure gradient varied inversely with fracture width on the order between 1 and 2, depending on the power-law index of the material. The power-law model also generated relations between pressure gradient and shear stress, and between superficial velocity and shear rate to bridge the gap between gel behaviors in fractures and in a rheometer. The power-law model explained why extrusion results strongly parallel the rheology measurement.
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LIST OF ACRONYMS AND SYMBOLS

CSR, controlled (steady) shear rate
CSS, controlled (steady) shear stress
HPAM, hydrolyzed polyacrylamide

Nomenclature

\( A = \) area, \( m^2 \)
\( c = \) consistency
\( f_B = \) Bingham yield point, Pa
\( f_C = \) Casson yield point, Pa
\( f_HB = \) Herschel-Bulkley yield point, Pa
\( F = \) force, N
\( F_{N_1} = \) normal force, N
\( G = \) elastic modulus, Pa
\( G' = \) storage modulus, Pa
\( G'' = \) loss modulus, Pa
\( h = \) gap height or fracture height, m
\( k = \) consistency index
\( L = \) fracture or tube length, m
\( m = \) Herschel-Bukley viscosity, Pa-s
\( M = \) torque, N-m
\( N = \) normal stress difference, Pa
\( N_1 = \) first normal stress differences, Pa
\( N_2 = \) second normal stress differences, Pa
\( n, p = \) power index
\( p_L = \) outlet pressure, Pa
\( p_0 = \) inlet pressure, Pa
\( \frac{dp}{dl} = \) pressure gradient, psi/ft [Pa/m]
\( q = \) volumetric flow rate, \( m^3/s \)
\( R = \) tube radius, mm
\( r = \) radius to tube wall, mm
\( t = \) time, s
\( T = \) normal stress, Pa
\( T_{11} = \) normal stress in the flow direction, Pa
\( T_{22} = \) normal stress perpendicular to the flow direction, Pa
\( T_{12}, T_{21} = \) shear stress, Pa
\( U_R = \) slip velocity, m/s
\( v = \) superficial velocity, m/s
\( v_s = \) effective slip velocity, m/s
\( w = \) aperture/fracture width, mm
\( w_f = \) fracture width, mm
\( x = \) distance to the fracture wall, mm
\( \alpha = \) cone angle, degree
\( \delta = \) phase angle, degree
\( \gamma = \) shear strain, \%
\( \gamma_0 = \) maximum shear strain, \%
\( \dot{\gamma} = \) shear rate, \(s^{-1}\)
\( \dot{\gamma}_0, \dot{\gamma}_{ar} = \) apparent shear rate, \(s^{-1}\)
\( \dot{\gamma}_R = \) corrected (actual) shear rate, \(s^{-1}\)
\( \dot{\gamma}_{Rw} = \) corrected apparent wall shear rate, \(s^{-1}\)
\( \eta = \) viscosity, Pa-s
\( \eta_B = \) Bingham viscosity, Pa-s
\( \eta_C = \) Casson viscosity, Pa-s
\( \eta^* = \) complex viscosity, Pa-s
\( \eta(\dot{\gamma}_R) = \) corrected viscosity, Pa-s
\( \tau = \) shear stress, Pa
\( \tau_0 = \) maximum shear stress, Pa
\( \tau' = \) in-phase shear stress, Pa
\( \tau'' = \) out phase shear stress, Pa
\( \tau_0' = \) in-phase maximum shear stress, Pa
\( \tau_0'' = \) out phase maximum shear stress, Pa
\( \tau_R = \) given shear stress, Pa
\( \tau_y = \) yield stress, Pa
\( \omega = \) angular frequency, \(s^{-1}\)
CHAPTER 1. INTRODUCTION

1.1 BACKGROUND

Excess water production increasingly plagues oil and gas production worldwide. In naturally fractured reservoirs, large volumes of Cr(III)-acetate-HPAM gels have been successfully extruded into place to reduce channeling and excess water production [1-4]. During gel placement in fractures, the pressure gradient for gel extrusion depends strongly on fracture width and gel composition [5-7]. Extrusion experiments directly measure gel properties in fractures, but they are both expensive (for core materials and casting) and time-consuming (two to three days per experiment, with one to two weeks of setup time). In contrast, a single rheology test can be performed at a fraction of the cost (negligible cost for gel materials) and can be completed within several minutes to one hour (after one to two days of setup time). Therefore, we investigated whether rheology testing can be used for predicting gel behavior in fractures.

It has been demonstrated that there are analogies of gel deformation between two parallel plates and in a fracture [8]. Earlier researchers [8-18] employed rheology measurements to characterize gelants and gels used in water shutoff systems. They used dynamic viscometry measurements to monitor gelation and studied gel properties after gelation. They found that Cr(III)-HPAM gels behaved as Bingham plastics under steady shear, withstanding shear stress up to a yield point, and then exhibiting a linear relation between shear stress and shear rate above the yield stress. The small-amplitude oscillatory measurements were found to cause minimum disruption to the gel structure.
and were therefore a reliable method for studying gel properties. They suggested that the measurement of elastic modulus ($G'$) should be used in place of bottle “tonguing” tests as a more quantitative measure of gel strength. In these studies, the viscosities, elastic modulus, and viscous modulus were related to the deformation frequency, the crosslinker density, the polymer and crosslinker concentrations, the polymer molecular weight, the temperature, and the aging time. Gels were susceptible to shear degradation during steady shear experiments. For the Cr(III)-HPAM gels and Cr(III)-Xanthan gels, the elastic components of viscosity need to be much greater (~100 times) than the viscous components. A simple force balance was applied to develop an equation that might relate the gel yield stress to the maximum pressure drawdown that a gel could withstand in the fracture [19].

1.2 PROBLEM IDENTIFICATION

1.2.1 Discrepancy between the Simple Force Balance Approach and the Extrusion Experiment for the Relationship of Pressure Gradient versus Fracture Width

Discrepancies were noted in using rheometers to predict gel behavior in fractures. First, a simple force balance predicted that the pressure gradient required for gel extrusion between two parallel plates should be inversely proportional to the open width between the plates [19]. However, the results from extrusion experiments showed that when gels extruded through fractures, the pressure gradient was roughly inversely proportional to the square of the fracture width [5-7].

The first challenge was to understand why the pressure gradient required for gel extrusion through an open channel was inversely proportional to the channel width for
the simple force balance approach, but was inversely proportional to the square of the fracture width in the extrusion experiments.

1.2.2 The Simple Force Balance Approach Underestimated the Pressure Gradient Required for Gel Extrusion through the Fracture

Using the force balance approach and the yield stress to estimate the pressure gradient required for gel extrusion through fractures, Liu [18] found that the pressure gradient was only about 1/87 of the experimental value. In Figure 1.1, Seright [5] plotted how the pressure gradient increased with HPAM concentration. It is obvious from the figure that the pressure gradients predicted by the simple force balance approach were much lower than those observed in the extrusion experiments.

![Figure 1.1—Pressure gradient and elastic modulus versus HPAM concentration [5].](image-url)
The second challenge was to investigate why the pressure gradients observed during gel extrusion through fractures were much greater (by more than 10 times) than those predicted from the simple force balance approach.

1.2.3 Similar Slopes for Complex Viscosity versus Shear Rate and Resistance Factor versus Injection Velocity

During steady shear measurements, Liu [8,18] found that for the 1X Cr(III)-acetate-HPAM gel, shear stress was fairly insensitive to shear rate (shown in Figure 1.2). When the shear rate was increased by a factor of $10^5$, the shear stress increased only by a factor of 10. This result was consistent with Seright's findings that the pressure gradient required to extrude gel through a fracture was fairly insensitive to injection velocity. A log-log plot of complex viscosity versus shear rate gave a slope of $-0.8$, which was close to the slopes ($-0.83$ to $-0.95$) for a log-log plot of resistance factor (apparent viscosity relative to water) versus injection velocity (shown in Figure 1.3) that were noted during gel extrusion through fractures [22, 23].

The third and final challenge was to understand the interesting similarities between the rheology measurements and the extrusion experiments.

1.3 RESEARCH OBJECTIVES

The purpose of this research is to determine whether rheology measurements would be a good substitute for the extrusion experiments. The objectives of this work are 1) to investigate why the simple force balance approach underestimated the pressure gradient required for gel extrusion through the fracture, 2) to develop a relationship
0.5% HPAM (5 million Mw)  
0.0417% Cr(III)-acetate  
Aged at 41°C for 24 hours  
Measured at 20°C using MP31.

Figure 1.2—Stress and complex viscosity versus shear rate [18].

24-hr-old Cr(III)-acetate-HPAM gel, 41°C

Figure 1.3—Resistance factor versus injection velocity [22, 23].
between the pressure gradient and the fracture width to solve the discrepancy, and 3) to understand the similarities between the rheological measurement (shear stress versus shear rate) and the extrusion experiment (pressure gradient versus flow rate).

1.4 THESIS ORGANIZATION

This dissertation has six chapters. Chapter 1 introduces the background information of this study and identifies the questions addressed in this work. Chapter 2 provides a brief look at the basic rheology and the literature that relates to this research. Chapter 3 describes the rheometer, the materials, and the experimental procedures that were used in this work. Chapter 4 discusses the rheological results of the effects of wall slip and first normal stress difference. Chapter 5 develops and analyzes a model based on the shell momentum balance and power-law equations. Chapter 6 summarizes the conclusions and raises questions concerning future work. The Appendix describes the development of the power-law model.
CHAPTER 2. THEORY AND LITERATURE REVIEW

2.1 BASIC RHEOLOGY

Rheology comes from the Greek verb, πέρπντο, to flow; it is the study of flow and deformation. Materials are divided into three groups according to their rheology properties; they are elastic solids, viscous liquids, and viscoelastic materials. An elastic solid has a rigid structure. Under loading it stores energy as potential for recovering shape after the load is released. It follows Hooke's Law. For viscous fluid, no elastic deformation occurs when shear stress is applied; instead, the fluid flows, dispersing the applied force and energy as heat. It follows Newton's Law. The properties of many real materials lie between solid and liquid and are called viscoelastic materials. A viscoelastic sample will deform and flow under the influence of an applied shear stress, but when the shear stress is removed, the sample will slowly recover from some of the deformation. Either the elastic nature or the viscous nature of the material may dominate the performance, depending on the timescale over which stress is applied. Many factors can affect the flow/deformation behavior of a viscoelastic material, including type of load, degree of load, duration of load, temperature, pH, pressure, and electrical or magnetic field strength.

2.1.1 Definition of Terms

The Two-Plate Model shown in Figure 2.1 is often used to define basic rheology parameters. The upper plate with the shear area \( A \) is moved by the shear force \( F \), and
the extent of this deflection has the value of $s$, with a resulting velocity of $v$. The bottom plate in the figure is immovable ($s = 0$, $v = 0$). The sample is sheared in the gap, $h$, between the plates. Two conditions must be met to obtain an exact calculation of the rheological parameters: 1) the sample must adhere to both plates without the occurrence of slide or slip, and 2) the resulting flow must be a laminar flow in the form of layers without turbulent flow occurring.

![Figure 2.1—The Two-Plate-Model for shear tests.](image)

- **Shear stress:** $\tau = \frac{F}{A}$  \hspace{2cm} 2-1
- **Shear strain:** $\gamma = \frac{x}{h}$  \hspace{2cm} 2-2
- **Shear rate:** $\gamma' = \frac{v}{h}$  \hspace{2cm} 2-3
- **Viscosity:** $\eta = \frac{\tau}{\gamma'}$  \hspace{2cm} 2-4
2.1.2 Hooke’s Law

The simplest relation between force and deformation is Hooke’s Law, which describes the ideal elastic deformation behavior as

\[ \tau = G\gamma. \]  \hspace{1cm} 2-5

Here, force (\(\tau\)) is proportional to the deformation (\(\gamma\)); \(G\) is the constant of proportionality, defined as the elastic modulus, and it is an intrinsic property of a solid.

2.1.3 Newton’s Law

Ideal viscous flow behavior is described by Newton’s law as

\[ \tau = \eta\dot{\gamma}. \]  \hspace{1cm} 2-6

In this equation, the stress (\(\tau\)) is proportional to the shear rate (\(\dot{\gamma}\)); the Newtonian viscosity (\(\eta\)) is the constant of proportionality. Gases and most small molecule liquids, like water and oils, are Newtonian.

2.1.4 Power-Law

We know that the viscosities of polymer solutions, polymer melts, glues, and shampoos decrease when shear rate increases; this behavior is called shear-thinning. Shear-thinning flow behavior is due to the three-dimensional coil structures of the polymer molecules which are entangled with each other at rest and disentangle to orient in the shear and shear gradient directions during the shearing process.

In contrast, the viscosities of ceramic suspensions, starch dispersions, and dispersions with a high concentration of solids or polymers increase with shear rate; this behavior is called shear-thickening. Shear-thickening flow behavior occurs in
concentrated, chemically unlinked polymers due to the mechanical entanglements between the mostly branched molecule chains. The higher the shear rate, the more the molecule chains prevent each other from moving. Similar behavior occurs with highly concentrated suspensions.

The most famous model function for shear-thinning and shear-thickening flow behavior is the Ostwald/de Waele, or power-law:

\[ \tau = c \cdot \dot{\gamma}^{n+1} \]

Here, \( c \) refers to consistency and \( n \) is the power-law index; \( n = 0 \) for Newtonian material; \( n < 0 \) for shear-thinning material; and \( n > 0 \) for shear-thickening material.

Flow behavior is displayed graphically using flow curves shown in Figure 2.2. The flow curve displays the mutual dependence of shear stress and shear rate, and the viscosity curve shows the relation of viscosity versus shear rate. Both shear stress and viscosity increase linearly with shear rate for Newtonian materials.

Figure 2.2—Flow curves of Newtonian and Power-law materials.
2.1.5 Yield Stress

Dispersions and gels have yield points due to intermolecular forces called van der Waals force. As an example, a certain amount of force must be applied before toothpaste can be squeezed out of the tube; this force is called the material's yield stress. The yield point is the point at which the material—toothpaste in this case—begins to flow as illustrated in Figure 2.3; it is the end of a reversible elastic deformation and the beginning of an irreversible deformation, or viscoelastic/viscous flow. In the low deformation range, the material exhibits a stable, solid-like structure which displays elastic behavior. But when the deformation exceeds the tolerance of the 3-D network, the structure is destroyed and the material flows.

Several mathematical models (e.g., the Bingham, Casson, and Herschel-Bulkley models) have been used to describe the flow curve function with apparent yield point [24].

Figure 2.3—Yield point.
2.1.5.1 Bingham Model

In the Bingham model,

\[ \tau = f_B + \eta_B \dot{\gamma}, \]  \hspace{1cm} 2-8

where \( f_B \) is called the Bingham yield point and \( \eta_B \) is the Bingham viscosity. The Bingham model is the simplest and was previously the most widely used model before flow curves were computer-analyzed. This model should only be used for the simplest of quality control checks.

2.1.5.2 Casson Model

In the Casson model,

\[ \tau^{1/2} = f_c^{1/2} + (\eta_c \dot{\gamma})^{1/2}, \]  \hspace{1cm} 2-9

where \( f_c \) is called the Casson yield point and \( \eta_c \) is the Casson viscosity. In this model, the transition from the Newtonian to the yield region is more gradual. For many materials, such as blood and food products, it provides a better fit.

2.1.5.3 Herschel-Bulkley Model

In the Herschel-Bulkley model,

\[ \tau = f_{HB} + m \dot{\gamma}^p, \]  \hspace{1cm} 2-10

where \( f_{HB} \) is called the yield point for the Herschel-Bulkley model and \( m \) is the Herschel-Bulkley viscosity. Compared with the Bingham model, the Herschel-Bulkley model also considers the shear-thinning and shear-thickening characteristics of the fluid above the yield point.
2.1.6 Viscoelastic Behavior

Many important materials, such as blood, polymers, and paint, lie between ideal elastic solid and ideal viscous fluid. Depending on the timescale over which stress is applied, either the elastic or the viscous nature of the material may dominate performance. Generally, the elastic nature dominates over short timescales, while the viscous nature becomes more evident over longer timescales. This time-dependent response is called viscoelasticity. Sinusoidal oscillation is a common method used to assess the viscoelastic nature of materials. When the sample is deformed sinusoidally, the stresses oscillate sinusoidally at the same frequency, but in general they will be shifted by a phase angle (δ) with respect to the strain wave. The applied strain (γ) is described as

\[ \gamma = \gamma_0 \sin(\omega t) \quad 2-11 \]

and the applied stress (τ) is

\[ \tau = \tau_0 \sin(\omega t + \delta) \quad 2-12 \]

Here, \( \gamma_0 \) and \( \tau_0 \) represent the maximum strain and stress, and \( \omega \) is the angular frequency of the strain wave, as shown in Figure 2.4. If the material is ideally elastic, the phase angle is zero. If the material is a Newtonian liquid, the phase angle is 90°. For viscoelastic materials, the stress wave will be shifted by an intermediate phase angle (e.g., the middle solid curve in Figure 2.4).

The stress wave can be separated into two waves (dashed curves in Figure 2.4) with the same frequency: \( \tau' \) for in-phase with strain wave and \( \tau'' \) 90° for out-of-phase with strain wave. In this way, the stress wave is separated into an elastic component (e.g., storage modulus, \( G' \)),

\[ G' = \frac{\tau'}{\gamma_0} \quad 2-13 \]
Figure 2.4—Sinusoidally oscillating shear strain produces a sinusoidal stress phase that is shifted by a phase angle [25].

and a viscous component (loss modulus, \( G'' \)),

\[
G'' = \frac{\tau_0}{\gamma_0}.
\]

The phase or "loss" angle (\( \delta \)) is related to \( G' \) and \( G'' \):

\[
\tan \delta = \frac{G''}{G'}.
\]

If the elastic modulus is larger than the viscous modulus, the material will exhibit a solid behavior; conversely, if the elastic modulus is larger than the viscous modulus, the material will behave like a liquid. The loss angle indicates whether it is the elastic or the viscous property that is dominant in the sample. For example, \( \delta = 0^\circ \) for an elastic solid, \( \delta \approx 1 - 5^\circ \) for hard rubber, and \( \delta \approx 15 - 30^\circ \) for soft rubber.

The complex viscosity, \( \eta^* \), is defined as a frequency-dependent viscosity function that is determined during oscillation.
2.2 LITERATURE REVIEW

2.2.1 The Simple Force Balance Approach Underestimated the Pressure Gradients Required for Gel Extrusion through the Fracture

When gel is extruded through a fracture, a minimum pressure gradient must be applied in order for the gel to flow, which suggests the gel material exhibits a yield stress [5–7, 20]. For a gel with yield stress under stationary (steady rate) conditions, Bird et al. [19] used a simple force balance to calculate the minimum pressure gradient required for gel to extrude through two parallel plates. Employing a UDS 200 rheometer with a smooth surface, parallel-plate geometry, Liu [18] estimated the yield stress of a 1X Cr(III)-acetate-HPAM gel in the range of 10 to 88 Pa. Using these values and an aperture of 1 mm (0.04 in), the pressure gradients predicted by the simple force balance approach were between 20 and 176 kPa/m (0.9 and 7.8 psi/ft). In contrast, the experimentally measured values ranged from 633 to 1,130 kPa/m (28 to 50 psi/ft) when the fracture width was 1 mm [7]. Therefore, the simple force balance approach greatly underestimated the pressure gradients required for gel extrusion through fractures.

Based on the previous research analysis, we considered four possible reasons for this discrepancy: wall-slip effect, first normal stress difference, gel dehydration, and wormholes in fractures.
2.2.1.1 Wall-Slip Effect

The experimentally measured pressure gradients were much greater (by more than 10 times) than those calculated from the simple force balance approach. In other words, the rheology-measured value of yield stress for our 1X Cr(III)-acetate-HPAM gel from Liu [18] was much lower than our calculated value from the simple force balance approach.

What factors might cause the measured values of yield stress to decrease? Macosko [25] mentioned that for viscoelastic materials, wall-slip effects were particularly prevalent during yield stress measurements. He noted that because of the impenetrability of the wall, the layer of particles adjacent to the rheometer wall was typically more dilute than the bulk dispersion. During flow, the shear rate gradient caused particles to migrate away from the wall. The thin, dilute layer near the wall had a much lower viscosity, creating the impression that the bulk fluid was slipping along the wall. Since the yield point was a flow/no-flow point as well as a transition point from elastic solid to viscous liquid, the existing thin, dilute layer acted as if the yield point had been reached and the material had already started to flow. The measured yield stress, in this case, was lower than its actual value. When measuring rheology properties using a smooth-surface geometry, wall-slip effects could exist and diminish the measured yield stress [8, 16–18]. Therefore, wall-slip effects must be identified and eliminated to ensure that the measured results are properties of the gels and not artifacts produced by the method of investigation.

The simplest way to identify the existence of the wall-slip effect is to compare the viscosity functions determined by capillaries having similar L to R ratios, but different R values; the smaller the capillary diameter, the greater effect of wall-slip, which results in
lower apparent viscosities. Mooney [26] noted that it was usually possible to correct the apparent wall slip and determine the true viscosity of the sample by extrapolating to infinite diameter:

\[ \dot{\gamma}_a = \dot{\gamma}_{\infty} + \frac{4v_s}{R}. \]  

Here, \( v_s \) is the effective slip velocity, \( \dot{\gamma}_a \) is the apparent wall shear rate, and \( \dot{\gamma}_{\infty} \) is the corrected apparent wall shear rate. Early researchers [9–10, 27–34] studied wall-slip effects in parallel-plate and couette measurement systems with suspensions, foams, highly concentrated emulsions, drilling mud, and pasty materials. Their results showed that 1) wall-slip effects could be identified by comparing stresses under certain shear rates with different gap heights (the distance between the parallel plates), and 2) wall-slip effects could be eliminated by adopting rough surfaces. Liu [8] demonstrated the existence of wall-slip effects of our 1X Cr(III)-acetate-HPAM gel when it was measured using smooth-surface, parallel-plate geometry by showing that the viscosity of the gel increased with gap.

In our research, we used the method outlined by Yoshimura and Prud’homme [35–36] to analyze wall-slip effects. Two assumptions were made when using this method: 1) that wall layer thickness was small compared to the viscometer gap, and 2) that slip velocity was only a function of stress once steady state was achieved. Yoshimura and Prud’homme measured stress versus shear rate with two different gap heights, \( h_1 \) and \( h_2 \). They found that if wall slip did not exist, the sample was purely sheared and the rheometer gap size would not play any role in the measurements. On the other hand, if wall slip occurred, the apparent viscosity increased with greater gap height. As gap height increased, the thin lubricating layer that was responsible for slip became less important.
According to Yoshimura and Prud'homme, the actual (corrected) shear rate at a given stress ($\tau_R$) is

$$
\dot{\gamma}_R (\tau_R) = \frac{h_1 \cdot \dot{\gamma}_{aR1} (\tau_R) - h_2 \cdot \dot{\gamma}_{aR2} (\tau_R)}{h_1 - h_2},
$$

with slip velocity

$$
u_s (\tau_R) = \frac{\dot{\gamma}_{aR1} (\tau_R) - \dot{\gamma}_{aR2} (\tau_R)}{2 \cdot \left( \frac{1}{h_1} - \frac{1}{h_2} \right)},
$$

and with corrected viscosity

$$
\eta(\dot{\gamma}_R) = \frac{(h_1 - h_2) \cdot \tau_R}{h_1 \dot{\gamma}_{aR1}(\tau_R) - h_2 \dot{\gamma}_{aR2}(\tau_R)},
$$

where $\dot{\gamma}_{aR1} (\tau_R)$, $\dot{\gamma}_{aR2} (\tau_R)$ are the apparent shear rates at a given shear stress.

Yield stress can be determined from a flow curve with preset shear rate using an approximation function (e.g., the Bingham, Casson, Herschel/Bulkley models or with the polynomial model). The yield point is determined by extrapolating the curve course to the shear rate value $\dot{\gamma} = 0$ as the intersection point on the $\tau$-axis.

The classical method of measuring yield stress uses a preset shear stress in the form of a ramp; the highest shear stress value at which the measuring instrument still detects no movement is taken as the yield stress. Yield point is the last measuring point at which the shear rate is still displayed as zero.
Since the yield point is also the limit of the elastic deformation range, Paar Physica [24] mentioned that an amplitude sweep could be used as an alternative way to measure yield stress.

Liu [18] measured the yield stress of the 1X Cr(III)-acetate-HPAM gel and got three possible values for yield stress in different ranges of shear rate, as shown in Figure 2.5, for this material.

![Figure 2.5—Different yield stresses in different ranges of shear rate for 1X gel [18].](image)

2.2.1.2 First Normal Stress Difference

Due to the anisotropic microstructures or forces in the interior of the viscoelastic material, when gels are sheared in fractures, normal stress difference occurs and exerts extra pressure on the walls, which may produce extra pressure gradient along the fracture. Previous researchers studied first normal stress difference in the form of extrudate, or die swell [37–39] (i.e., the expansion that occurs when a viscoelastic liquid leaves a capillary die), or as the Weissenberg effect [40–41] (i.e., during stirring, viscoelastic liquids creep
up the stir shaft—also known as stir-rod climbing). In these previous studies, first normal stress difference was measured in steady shear flows [37, 42–46] or was predicted by models [37–39, 47]. For polymers, the steady shear response for the first normal stress difference, \( N_1 \), usually exhibits the maximum slope at low shear rates and then gradually flattens out as shear rate increases [44].

Both the die swell and Weissenberg effects are related to first normal stress difference. With a simple viscous fluid, only the resistance force \( (F) \) or the shear stress \( (\tau) \) occurs and acts in the direction of flow, but no force acts perpendicular to the wall. With a viscoelastic material, a normal force \( (F_N) \) can occur that presses the two shearing areas apart. The normal stress \( (T) \) occurs due to anisotropic microstructures present in the viscoelastic material.

When a viscoelastic material is sheared between two parallel surfaces (Figure 2.6), two normal stress differences are produced in addition to the viscous shear stress; these two normal stress differences are \( N_1 = T_{11} - T_{22} \) and \( N_2 = T_{22} - T_{33} \). The subscripts are defined as follows: “1” is the flow direction, “2” is the direction perpendicular to the surfaces between which the fluid is sheared, and “3” is neutral. The larger of the two normal stress differences is \( N_1 \), and it is responsible for the rod-climbing phenomenon. For isotropic materials, \( N_1 \) is positive in sign (unless it is zero). In a rheometer with cone-plate geometry, \( N_1 \) causes the cone and plate surfaces to be pushed apart; \( N_2 \) is usually found to be negative and smaller in magnitude than \( N_1 \). At sufficiently low shear rates, shear stress \( (T_{12}) \) usually becomes linear in the shear rate \( (\dot{\gamma}) \) (i.e., the shear viscosity,
Based on the above analysis, the first normal stress difference should be very small when the shear rate is very low (normally, below 0.1 s^-1). In this case, the effect of normal stress is negligible. However, in our extrusion experiments, the injection rates were between 0.046 and 23.4 cm/s. For a 1-mm wide fracture, the maximum shear rates were between 0.92 and 468 s^-1, which exceed the range for low shear rates. Steady shear measurements were therefore needed to determine the first normal stress difference at higher shear rates.

2.2.1.3 Gel Dehydration

Before gelation, fluid gelant solutions can readily leak off from fractures into porous rock. However, after gelation, the crosslinked materials will not penetrate significantly into the porous rock [5-7]. Cr(III)-acetate-HPAM gels (as well as other gels) concentrate or lose water during extrusion through fractures. Thus, gels dehydrate
during extrusion through fractures. The dehydrated gel had a higher concentration of polymer and crosslinker than injected gels. On average, it contained 12 times the polymer concentration of the injected gel.

Liu [8] found that the elastic modulus increased with the polymer concentration, so yield stress should also increase with polymer and crosslinker concentrations. A reasonable assumption would be that more concentrated gels might have higher yield stress and might result in higher pressure gradients than the injected 1X Cr(III)-acetate-HPAM gel.

2.2.1.4 Wormholes in Fractures

Another possible reason that the simple force balance approach might have underestimated the pressure gradient relates to the size of the flow channel. The fracture width was used as the opening path width in the simple force balance approach. However, in reality, flow channels for gels could be much smaller than the fracture width due to the “wormhole” model suggested by Seright [5]. As we know, Cr(III)-acetate-HPAM gels (as well as other gels) concentrate or lose water during extrusion through fractures. As water leaks off from the gel, the gel concentrates (by factors up to 50) to become immobile in the vicinity where dehydration occurs. The driving force for gel dehydration (and water leakoff) is the pressure difference between the fracture and the adjacent porous rock. Because fresh gel (i.e., mobile gel, with the original composition) is much more pliable and mobile than the concentrated gel, the fresh gel wormholes through the concentrated gel in order to advance the gel front [5–7].
These wormholes may be significantly narrower than the fracture width. This possibility could explain the relatively high pressure gradients during gel extrusion. In the wormhole pattern, shown in Figure 2.7, the dark areas indicate the mobile gel and the light areas show the immobile gels.

2.2.2 The Relationship of Pressure Gradient versus Fracture Width

2.2.2.1 The Simple Force Balance Approach

For a Bingham material with yield stress ($\tau_y$) under stationary (steady rate) conditions, Bird et al. [19] used a simple force balance to calculate the minimum pressure gradient ($dp/dl$) required for the material to extrude through two parallel plates separated...
by a distance of \( w \) (Figure 2.8):

\[
dp/dl = \frac{2\tau_y}{w}.
\]  

(2-21)

The simple force balance suggested that the minimum pressure gradients required for gel extrusion through fractures should be inversely proportional to the fracture width or capillary radius. In a capillary of radius, \( R \),

\[
dp/dl = \frac{2\tau_y}{R}.
\]  

(2-22)

2.2.2.2 Extrusion Experiments

Ganguly et al. [23] demonstrated that rupture pressure was inversely proportional to the inside diameter of the tube when gels (0.75% Alcoflood 935, 0.0417% Cr(III) acetate, 1% NaCl; gel time = 12 hours, gelant aged 6 to 8 days) were placed in various lengths of nylon and polypropylene tubes with different inside diameters. Seright [20] noted similar results when studying failure of the 1X gel in tubes made of stainless steel,
Teflon, glass, and polyetheretherketone. A power regression on the data confirmed that the failure pressure gradient was inversely proportional to the tube diameter. However, the correlation coefficient for the regression was only 0.6.

Seright [6, 20] also noted that the pressure gradient required for gel extrusion was inversely proportional to the square of the fracture width (Figure 2.8). For our 1X Cr(III)-acetate-HPAM gel, the required pressure gradient ($dp/dl$, in psi/ft) could be estimated using Equation 2-23 if fracture width ($w_f$) is expressed in inches:

$$\frac{dp}{dl} = 0.02 \left( \frac{w_f}{in} \right)^2$$  \hspace{1cm} 2-23

2.2.2.3 The Bingham Model

To describe gel behavior in the fracture, Seright [5] examined a Bingham model for extrusion of the Cr(III)-acetate-HPAM gels through fractures. In this model, a Newtonian fluid near the fracture wall lubricated the flow of the solid-like materials in the middle of the path. Water leaving the gel during the dehydration process could comprise the lubricating layer. The thickness of the lubricating layer should be small, relative to the fracture width, and increase linearly with fluid velocity.

Two flaws existed with the Bingham model. First, the pressure gradients predicted by this model should depend on the viscosity of the lubricating fluid—water in this case. Since the viscosity of water decreased by a factor of about three as temperature increased from 20° to 80°C, the measured pressure gradients for gel extrusion should also have decreased by roughly a factor of three. In reality, the pressure gradient was insensitive to temperature over this range. Second, the lubrication layer should have become thicker with increased flow rate. With a thicker lubrication layer (i.e., more
water), the leakoff rate should have increased with the increased extrusion rate. Instead, the leakoff rate was independent of flow rate—it was primarily time-dependent [5–6].

Liu [8, 18] noted that the rheology behavior of Cr(III)-acetate-HPAM gels was better described using the Herschel-Bulkley model. Neither the simple force balance approach nor the Bingham model considered the shear-thinning property of gel materials when shear stress was above the yield point. In view of these deficiencies, we considered an alternative model that was based on the Herschel-Bulkley model [25], where the fluid exhibited a yield stress and flowed as a power-law fluid above the yield point. We employed a shell momentum balance [25] and power-law equations to deduce these models.

### 2.2.3 Similarity between Rheology Measurement (Shear Stress versus Shear Rate) and Extrusion Experiment (Pressure Gradient versus Injection Rate)

During steady shear measurements, Liu [8, 18] found that for the 1X Cr(III)-acetate-HPAM gel, shear stress was fairly insensitive to shear rate. When the shear rate was increased by a factor of $10^5$, the shear stress increased only by a factor of 10. This result was consistent with Seright's findings that the pressure gradient required to extrude gel through a fracture was fairly insensitive to injection velocity [22, 23]. A log-log plot of complex viscosity versus shear rate gave a slope of $-0.8$, which was close to the slopes ($-0.83$ to $-0.95$) for a log-log plot of resistance factor (apparent viscosity relative to water) versus injection velocity that were noted during gel extrusion through fractures [22–23]. This interesting similarity made us consider the relationship of gel behavior in fractures versus in a rheometer.
As we know, viscosity is a function of shear stress, and resistance factor (apparent viscosity relative to water) is a function of pressure gradient. Therefore, the relationship between complex viscosity and shear rate could be considered as shear stress versus shear rate. The relationship between resistance factor and injection velocity could be considered as pressure gradient versus injection velocity. If we can correlate pressure gradient versus shear stress and injection velocity versus shear rate, we can convert pressure gradient and injection velocity from the extrusion experiments into shear stress and shear rate. When the calculated shear stress and shear rate are compared with the rheology measurement, a bridge is created between the extrusion experiment and the rheology test.
CHAPTER 3. EXPERIMENTATION

3.1 RHEOMETER, PHYSICA UDS 200

The PHYSICA UDS 200 Universal Dynamic Spectrometer shown in Figure 3.1 is a drag-flow rheometer in which shear is generated between a moving and a fixed solid surface.

![PHYSICA UDS 200](image)

**Figure 3.1—PHYSICA UDS 200.**

This spectrometer is used for research and development as well as for quality control. The unit is designed to measure the viscosity of Newtonian fluids, to determine the viscosity function of non-Newtonian fluids at stationary shear flow, and to measure viscoelastic quantities occurring during oscillatory, creep, strain, and relaxation tests.

The PHYSICA UDS 200 rheometer system consists of the following:
• MC 200 electronics—digital control of rotational speed and torque
• UM 200 measuring drive system
  o Air-bearing measuring drive with quick-fitting coupling for the measuring system and for extremely wide torque range
  o Microprocessor-controlled motor electronics for setting measuring gaps to an accuracy of less than 1 μm
  o Interchangeable temperature control systems
• Software US 200, developed for Microsoft Windows Version 3.1 or better, controls the UDS 200 systems
• A thermostat applies accurate temperature control—this is essential for obtaining reliable and reproducible data
• Printer / plotter

Although the PHYSICA UDS 200 has four different measuring systems (i.e., cone-plate, parallel-plate, concentric-cylinder, and double-gap cylinder geometries), we used only cone-plate and parallel-plate measuring systems for the rheology tests performed in this study.

3.1.1 Cone-Plate Geometry

The cone-plate measuring system is used for all standard tests and for measuring first normal stress difference. This measuring system uses a bob with a conical surface and a plate with a flat surface as shown in Figure 3.2. The cone-shaped bob connects to a motor that rotates at various speeds and the plate connects to a torque-measuring device that measures the plate’s resistance when the cone is rotated at different speeds; the test
sample is trapped between the cone and the plate. The flow curve for the sample is generated by measuring the plate's torque as the cone rotates at different speeds.

![Cone-plate geometry measuring system.](image)

The advantages to using cone-plate geometry [48] are as follows:

- A constant shear rate is maintained throughout the sample.
- Only a small amount of sample is required.
- Most of the air bubbles present are pressed out of the conical shear gap prior to testing.
- Loading the sample and cleaning the apparatus is easy.

However, cone-plate geometry also has two distinct disadvantages [48]:

- The sample can evaporate rather easily.
- Its particle size cannot be greater than 10 μm.

The geometry of the cone is determined by cone radius (R) and cone angle (α). A small cone angle (α ≤ 4°) is recommended to ensure a constant shear rate from the cone
tip to the outer radius of the cone. For low-viscosity fluids, large cone and plate radii are preferred; for high-viscosity samples, a small radius fixture is used. Four cone-plate geometries with different radii and cone angles come with the PHYSICA UDS 200: MK 20 (12.5 mm, 1°), MK 21 (12.5 mm, 2°), MK 22 (25 mm, 1°), and MK 23 (25 mm, 2°). MK 22 was selected for measuring first normal stress difference due to the viscosity ranges of the gel materials in this study.

For both Newtonian and non-Newtonian fluids, the basic equations for the cone-plate measuring system [48] are:

\[ \tau = \frac{3M}{2\pi R^3} \]  
\[ \dot{\gamma} = \frac{\omega}{\alpha} \]  
\[ \eta = \frac{3M\alpha}{2\pi R^3 \omega} \]

Here, \( \tau \) is the shear stress, \( \dot{\gamma} \) is the shear rate, and \( \eta \) is the shear viscosity; \( M \) is the torque, \( \omega \) is the angular frequency, \( \alpha \) is the cone angle, and \( R \) is the cone radius.

### 3.1.2 Parallel-Plate Geometry

This measuring system consists of two identical, coaxial, parallel plates; both plates are flat, as shown in Figure 3.3. The sample is loaded in the gap \( (h) \) between the two plates, and the system requires that \( h \leq R \).

Parallel-plate geometry can be used for all of the standard tests and for normal force measurements. Its two most obvious advantages over cone-plate geometry are that

- samples containing relatively large particles can be measured, and
- shear rate can be easily varied by changing the gap \( (h) \).
Compared to cone-plate geometry, its disadvantages are that

- data analysis can become complicated due to lack of constant shear gradient in the gap since shear rate increases in value from zero at the center of the plate to maximum at the edge, and
- shear rate and shear stress are not homogeneous in the gap; as a result, spreading and leakoff of the sample can occur at the edge of the plate.

The geometry of the plate is determined by plate radius ($R$). Generally, it is recommended that a large plate be used for low-viscosity fluids and a small plate for high-viscosity, stiff, or rigid samples. The PHYSICA UDS 200 comes with three parallel-plate measuring systems with different radii: MP 30 (12.5 mm), MP 31 (25 mm) and MP 32 (37.5 mm). In this study, MP31 was used for all steady shear and dynamic oscillation tests, except for first normal stress difference measurements.
In the steady shear tests, shear stress and shear rate are calculated at the edge of the plates \((r = R)\) [48]:

\[
\dot{\gamma}(R) = \frac{\omega R}{h}
\]

\[
\tau(R) = \frac{3M}{2\pi R^3} \left( 1 + \frac{1}{3} \frac{d \ln M}{d \ln \dot{\gamma}(R)} \right)
\]

3.2 MATERIALS

The polymer used for all the experiments was a partially hydrolyzed polyacrylamide (HPAM): Ciba Alcoflood 935™ with a molecular weight of \(5 \times 10^6\) daltons and 5–10\% degree of hydrolysis. Chromium acetate was used as the crosslinker and sodium chloride and calcium chloride were used to obtain the desired salinity. First, 1\% NaCl and 0.1\% CaCl\(_2\) were added to distilled water to make brine; the polymer was then carefully added to the brine with continuous stirring to avoid clumping. It is essential to maintain an optimum stirring rate when adding polymer to the saline solution. Parafilm was used to seal the beaker prior to placing it on the stirrer; stirring continued until all the polymer particles were dissolved in the brine. Finally, the crosslinker was added to the polymer solution and stirring continued until the solution became homogeneous. The sample was then sealed and aged for 24 hours at 41°C.

Our standard 1X, 2X, and 3X gels were used in the measurements. The 1X gel contained 0.5\% Alcoflood 935 HPAM, 0.0417\% Cr(III) acetate, 1\% NaCl, and 0.1\% CaCl\(_2\). The multipliers of X refer to the HPAM and chromium acetate concentrations relative to those used in our standard 1X gel. In all cases, the HPAM/Cr(III)-acetate ratio was fixed at 12:1.
3.3 RHEOLOGY MEASUREMENTS

All the rheology measurements were performed at 41°C in order to compare the rheology data with the results from the extrusion experiments.

3.3.1 Steady Shear Tests with Controlled Shear Rate

First, steady shear measurements with controlled shear rate (CSR) were performed using different gap heights and various rough surfaces to identify the existence of wall-slip effects and to compare the results of wall-slip prevention measures. The parallel-plate geometry, MP31 (50 mm), was used for the CSR steady shear test. For the first set of tests, the gap height between the plates was set at $h_1 = 0.5$ mm. The upper plate was rotated at a fixed shear rate and the resultant stress measurement on the bottom plate was recorded. This test was repeated for a range of shear rates to obtain the flow curve (i.e., stress versus shear rate). For the next set of tests, the gap height was increased to $h_2 = 1$ mm and the above processes were repeated to obtain another flow curve. Smooth curves were fit through the points and the actual (corrected) shear rate was calculated using Equation 2-18. These procedures were performed on smooth, sandblasted, profiled, and sandpaper-covered surfaces for our 1X, 2X, and 3X gels.

Steady shear tests with controlled shear rate (CSR) were also performed to measure the first normal stress difference. The cone-plate geometry, MK 22 (25 mm, 1°), was used to measure first normal stress difference. Normal force ($F_N$) was preset as zero prior to loading the sample; the sample was then loaded and a wait period ensued until the normal force returned to the original preset value, or a stable value close to zero. The shear rate was preset to increase stepwise from 0.005 to 1000 s$^{-1}$, depending on the material, and the rheometer recorded the resultant first normal stress difference. The time
to reach steady state ranged from about 0.1 to 100 s, depending on the shear rate: the smaller the shear rate, the longer the time required to reach steady state. Measurements of first normal stress difference were performed using our 1X, 2X and 3X gels.

3.3.2 Oscillatory Tests

An oscillation is defined by two variables. The first variable is the amplitude, which is the maximum height of the oscillation wave; the second variable is frequency, which is the number of repeating periods. Amplitude and frequency sweeps were performed to verify the existence of wall-slip effects, to measure the dynamic properties of the gels, and to determine the yielding area. The parallel-plate geometry, MP31 (50 mm), was used for the oscillation tests. Strain amplitude sweeps were performed at a constant frequency of 1 Hz over the strain range of 1–10\(^5\)% to obtain the storage modulus, loss modulus, complex viscosity, and breaking strain. The breaking strain, \(\gamma_b\), was defined as the strain at which the storage modulus fell below 90% of its plateau value. As the strain increased, the gel structure was destroyed, resulting in a decrease in the storage modulus; the loss modulus increased with strain until it reached the maximum value and then decreased. These behaviors have been associated with materials having yield stresses [49]. Frequency sweeps were conducted at 10% strain to ensure linear viscoelastic response over the frequency range from 0.1–10 Hz. Frequency sweep was used to compare storage modulus, loss modulus, and complex viscosity with values obtained from the amplitude sweep. These measurements were conducted on smooth, sandblasted, profiled, and sandpaper-covered surfaces with our 1X, 2X, and 3X gels.
3.3.3 Creep Tests

Ideal elastic materials display an immediate reformation after loading and subsequent removal of the load. A similar elastic behavior is seen in viscoelastic samples after a certain time delay. Creep tests were performed to determine the retardation time which describes the delayed reaction to an applied force or stress. In this study, creep tests were performed to measure the yield stress. All the measurements were conducted using the parallel-plate geometry, MP31 (50 mm), with a double-rough surface (profiled-sandpaper surface or sandpaper-sandpaper surface); 1X, 2X, and 3X gels were tested to determine their yield stresses. The stress was incremented stepwise from 10 to 500 Pa depending on the different gels used, and the resultant shear rate was recorded. When shear stress was applied above the yield stress, the gels typically reached steady state in less than 10 seconds. Yield stress values were defined as the lowest points at which stress produced steady state deformation. The time required for each step ranged from about 10 to 100 seconds.

3.4 EXTRUSION EXPERIMENTS

We used the results from Seright’s [50–53] extrusion experiments for comparison with our rheology measurements in order to evaluate the effectiveness of using rheology measurements to estimate gel behavior in fractures.

The cores used in these experiments were Berea sandstone. Each core measured 48 X 1.5 X 1.5 inches and had four internal pressure taps. After casting in epoxy, the fractured cores were saturated with brine to determine their porosities and permeabilities. Next, 10 to 17 pore volumes (PV) of gel were injected into the fractured core at a rate of 200 ml/hr. Following the injection, the cores were shut in for several days and were then
re-injected with brine. The experiments were conducted at 41°C. After the extrusion experiments, the concentrated gels found on the wall of the fracture were peeled from the fracture wall and their rheological properties were investigated.
CHAPTER 4. EXPERIMENTAL RESULTS AND DISCUSSION

4.1 WALL-SLIP EFFECT ON THE PRESSURE GRADIENT

Wall-slip effects were analyzed using the method suggested by Yoshimura and Prud’homme [35]. Steady shear and oscillatory sweep measurements were taken using smooth parallel-plate geometries with various gap heights to identify the presence of wall-slip effects. Double-rough surfaces were employed to eliminate wall-slip effects and to measure the yield stresses without the effect of wall slip. The pressure gradients were calculated using the simple force balance approach and the yield stress and were then compared with the results from the extrusion experiments.

4.1.1 Steady Shear Tests for Flow Curve

Steady shear tests with controlled shear rate were performed on smooth and double-rough surfaces for the 1X, 2X, and 3X gels to identify and eliminate wall-slip effects.

4.1.1.1 Smooth surfaces

Figures 4.1, 4.2 and 4.3 show the flow curves plotted for 1X, 2X, and 3X gels using the smooth-surface rheometer at 0.5- and 1-mm gap heights. When the shear rate increased from 10 to 1000 s⁻¹, shear stress increased from 50 to 250 Pa for the 1X gel, and from 200 to 1000 Pa for the 2X gel. For 3X gel at shear rates between 1 and 1000 s⁻¹, shear stress increased from 350 to 2000 Pa. All the flow curves show decreasing slopes,
Figure 4.1—Flow curves for the 1X gel on smooth surfaces.

Figure 4.2—Flow curves for the 2X gel on smooth surfaces.
indicating that the viscosities decreased with shear rate; this phenomenon is defined as shear-thinning behavior.

The flow curves plotted for the 1X and 2X gels clearly show higher shear stresses at the 1-mm gap height (solid squares) than at 0.5 mm (open diamonds). The separation in the flow curves for these two gap heights suggest a wall-slip effect on the measurements. However, the flow curves for the 3X gel with gap heights of 0.5 mm and 1 mm overlapped, indicating that the wall-slip effect was negligible. The result for the 3X gel is consistent with the phenomenon that wall-slip effect was less important for more concentrated foams in the steady shear tests [34]).

For the 2X gel, a critical shear stress of about 300 Pa was observed. We noted a discrepancy between the two flow curves with different gap heights when shear stress was below the critical stress value, indicating that wall-slip effects were present when shear stress fell below 300 Pa. When shear stress rose above 300 Pa, the flow curves
overlapped one another and wall slip became negligible. This critical shear stress was considered as the yield stress for the 2X gel. When the shear stress was below the yield stress point, wall slip effects became significant; but when the bulk fluid began to flow, the effect of wall slip became negligible [27, 34]. The existence of critical shear stress was not obvious on the flow curves for the 1X and 3X gels.

4.1.1.2 Double-rough surfaces

If wall-slip effects occur during shearing, a measuring system with modified surfaces may be used in place of smooth and polished surfaces in order to achieve improved adhesion between the sample and the walls of the measuring system [48]. We tried different methods (e.g., making gel in the measuring system, using sandblasted-smooth-, profiled-smooth-, or sandpaper-covered-surfaces) to eliminate wall-slip effects. However, when making gel in the stainless steel measuring system the gelant did not undergo gelation. We do not know the reason for the failure; one possibility could be the materials used to fabricate this measuring system. With the measuring system made of titanium, it was difficult to control the gel thickness in the gap without damaging the gel structure. We also found that using a single rough surface (sandblasted-smooth or profiled-smooth) was not sufficient to eliminate wall slip because wall slip occurred on both the top and bottom plates. Therefore, we covered the top plate with sandpaper and used sandblasted, profiled, or sandpaper-covered bottom plates (i.e., the double-rough surface measuring system).

The steady shear tests were repeated on the double-rough surfaces with two different gap heights. Flow curves for the 1X, 2X, and 3X gels on different surfaces are shown in Figures 4.4, 4.5, and 4.6, respectively.
For the 1X and 2X gels, the shear stress values on the double-rough surfaces were greater than those on the smooth surface. The flow curves for the double-rough surfaces at two different gap heights were fairly close, signifying that wall-slip effects were significantly reduced by this surface. The flow curves for the 3X gel on various surfaces were not significantly different, indicating that wall-slip effects were negligible for this material.

![Flow curve comparison of 1X gel on different surfaces.](image)

Figure 4.4—Flow curve comparison of 1X gel on different surfaces.
Figure 4.5—Flow curve comparison of 2X gel on different surfaces.

Figure 4.6—Flow curve comparison of 3X gel on different surfaces.
Yoshimura and Prud'homme's method was used to correct for wall slip on the smooth surface; corrected values of shear rate were calculated using Equation 2-18. Corrected flow curves (solid lines) were generated for the 1X and 2X gels and compared with those for the double-rough surfaces shown in Figures 4.4 and 4.5. The corrected flow curves 1) overestimated the shear stress curves at low shear rates, indicating that wall slip had not been completely eliminated on the double-rough surface; and 2) underestimated the shear stress values at high shear rates, which might be due to the operating limitations of the rheometer. Another possible reason could be a result of the heterogeneous structure of the gel materials. Yoshimura and Prud'homme deduced the Equation 2-18 with the assumption that slip is the same at both plate surfaces; however, this may not be true for gels. More work is needed to understand this behavior.

4.1.2 Oscillatory Sweep for the Dynamic Properties

Oscillatory measurements were also performed to study wall-slip effects and to determine the viscoelastic property of the gels.

4.1.2.1 Amplitude Sweep

Amplitude sweeps were conducted using different gap heights (0.5 to 2 mm) and various surfaces to evaluate the presence of wall-slip effects. Figures 4.7, 4.8 and 4.9 compare storage and loss moduli for the 1X, 2X, and 3X gels on smooth and rough surfaces at various gap heights. The figures show the results for double-rough surfaces (solid diamonds and triangles) and smooth surfaces (open diamonds and triangles), as well as for storage (open diamonds) and loss (triangles) moduli. We also used trendline equations to compare how much storage and loss moduli increased with increasing gap height to compare wall-slip effects on different surfaces.
For the 1X gel, Figure 4.7 reveals the following points: 1) on smooth surfaces, both storage and loss moduli increased with gap height, indicating the presence of wall-slip effects; 2) on double-rough surfaces, storage and loss moduli were nearly independent of gap height, indicating that the wall-slip effects were reduced by the rough surfaces; and 3) storage and loss moduli on double-rough surfaces were larger than those measured on smooth surfaces, indicating that wall-slip effects were reduced by double-rough surfaces. These conclusions were consistent with observations from the steady shear tests. Similar studies using the 2X and 3X gels were less definitive.

![Figure 4.7—Dynamic property comparison for the 1X gel versus gap on different surfaces.](image-url)
Figure 4.8—Dynamic property comparison for the 2X gel versus gap on different surfaces.

Figure 4.9—Dynamic property comparison for the 3X gel versus gap on different surfaces.
4.1.2.2 Frequency Sweep

Figures 4.10, 4.11, and 4.12 show the results of the frequency sweep tests performed on the double-rough surface to determine the dynamic properties for the 1X, 2X, and 3X gels. In the figures, solid diamonds and open diamonds represent storage modulus ($G'$) and loss modulus ($G''$), respectively.

In these tests the gels were only able to deform without becoming irreversibly deformed or even destroyed within a very narrow range of frequency. The frequency sweep curves describe the densely cross-linked polymer behavior—that storage and loss moduli are frequency-independent. A comparison of the storage and loss moduli for each of the gels showed that the storage modulus was much greater (by more than 10 times) than the loss modulus over the entire frequency range investigated.

![Dynamic moduli versus frequency for the 1X gel at 41°C.](image)

**Figure 4.10**—Dynamic moduli versus frequency for the 1X gel at 41°C.
Figure 4.11—Dynamic moduli versus frequency for the 2X gel at 41°C.

Figure 4.12—Dynamic moduli versus frequency for the 3X gel at 41°C.
The difference between the two moduli increased with increasing gel concentration, indicating that the gel materials were predominantly elastic in nature—a property of polyacrylamide gels that has been observed many times over by other researchers [55-58].

4.1.3 Stress Sweep for Yield Stress

Using the definition of yield point (i.e., the transition point between the elastic property and the viscous property of a material), Paar Physica [24] discussed how amplitude sweep could be used to measure yield stress. The measuring curves, $G'$ and $G''$ functions, are represented in the diagram over the shear stress. Notice that the yield point is the one shear stress value where the measuring curve deviates noticeably from the plateau value or from the draw analysis tangent. The bandwidth of the tolerated deviation must be defined by the user (e.g., as 1%, 5% or 10%). Pal [27] defined the yield stress of emulsions as the point above which storage modulus decreased by 10% of the constant value. We took 10% as the bandwidth of the tolerated deviation to determine the yield point.

Stress sweep tests were performed on the double-rough surface to determine yield stress values for the 1X, 2X, and 3X gels. Figures 4.13, 4.14, and 4.15 show that the average values for storage modulus (solid diamonds) and loss modulus (open diamonds) are consistent with the results from the amplitude and frequency sweep tests.

The stress sweep curves shown in the figures indicate that below certain critical values of shear stress, the storage modulus remained nearly constant as the shear stress increased. However, when the shear stress surpassed the critical value, there was a sharp
Figure 4.13—Yield stress for the 1X gel in stress sweep at 41°C.

Figure 4.14—Yield stress for the 2X gel in stress sweep at 41°C.
Figure 4.15—Yield stress for the 3X gel in stress sweep at 41°C.

decrease in the storage modulus, indicating that a breakdown had occurred in the gel’s microstructure. This critical shear stress was considered as the yield stress for the gel. Yield points (cross-section points) for each gel were marked on the curves. Critical shear stress values for the 1X, 2X, and 3X gels were around 20 Pa, 67 Pa, and 364 Pa, respectively. These values closely matched those obtained from the steady shear tests for flow curves.

4.1.4 Creep Tests for Yield Stress

Yield stress was determined for each of the gel materials without the effect of wall-slip. The simple force balance approach was used to calculate the pressure gradient required for gel extrusion through the fracture. Steady shear tests with controlled shear stress (CSS), also known as the creep test, were performed using double-rough surfaces to determine yield stress. When the shear stress falls below the yield stress, materials
cannot flow and the shear rate is zero. Once the applied shear stress exceeds the yield stress of the material, the material deforms continuously under constant shear rate.

Figures 4.16, 4.17, and 4.18 show the creep test results for our 1X, 2X, and 3X gels with double-rough surfaces using the parallel-plate measuring system at 41°C. The yield stress was identified when the strain-versus-time curves became sensitive to the applied stress. For example, for the 1X gel, the strain-versus-time curves were insensitive to the applied stress below 71 Pa; above 72 Pa, the strain-versus-time curves were significantly higher. When applied stress exceeded the values of yield stress, the slope of the diagrams of strain-versus-time with logarithmic scale were equal to one (i.e., constant shear rates were obtained). The yield stress was 72 Pa for the 1X gel, 141 Pa for the 2X gel, and about 400 Pa for the 3X gel; these results were close to the measurements from the stress sweeps.

![Creep test results](image)

**Figure 4.16**—Creep tests for yield stress of the 1X gel at 41°C.
Figure 4.17—Creep tests for yield stress of the 2X gel at 41°C.

Figure 4.18—Creep tests for yield stress of the 3X gel at 41°C.
Liu [8, 18] measured the yield stress of a 1X gel by tangent crossover on the flow curve. She listed three possible values for the yield stress, from 10 to 88 Pa, but had difficulty determining the actual yield point. The yield stress from our measurements on the 1X gel was closer to her highest value. The other two possible choices for the yield point from Liu’s work may have been influenced by wall slip.

4.1.5 Summary of Rheology Properties of 1X, 2X, and 3X Gels

Table 4.1 summarizes the rheology properties for our 1X, 2X, and 3X gels. The consistency index and power index were obtained by fitting the flow curves to the stress-versus-shear-rate values from steady shear with controlled shear rate (Figure 4.1, 4.2, and 4.3).

<table>
<thead>
<tr>
<th>Material</th>
<th>Yield Stress, ( \tau_{\mu}, \text{Pa} )</th>
<th>Consistency Index, ( k )</th>
<th>Power Index, ( n )</th>
<th>Storage Modulus, ( G', \text{Pa} )</th>
<th>Loss Modulus, ( G'', \text{Pa} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1X gel</td>
<td>72</td>
<td>41.5</td>
<td>-0.75</td>
<td>3.81</td>
<td>0.444</td>
</tr>
<tr>
<td>2X gel</td>
<td>141</td>
<td>219</td>
<td>-0.79</td>
<td>36.1</td>
<td>0.587</td>
</tr>
<tr>
<td>3X gel</td>
<td>400</td>
<td>537</td>
<td>-0.81</td>
<td>117</td>
<td>1.90</td>
</tr>
</tbody>
</table>

Flow behaviors for the 1X, 2X, and 3X gels were better described by the Herschel-Bulkley model [48]—the fluid exhibited yield stress and flowed as a power-law fluid for shear stress above the yield point:

\[
\text{1X gel: } \tau = 72 + 41.5 \dot{\gamma}^{0.25} \quad (\tau > 72 \text{ Pa})
\]

\[
\text{2X gel: } \tau = 141 + 219 \dot{\gamma}^{0.21} \quad (\tau > 141 \text{ Pa})
\]

\[
\text{3X gel: } \tau = 400 + 537 \dot{\gamma}^{0.19} \quad (\tau > 400 \text{ Pa})
\]
From Table 4.1, we concluded that using higher concentrations of polymer and crosslinker yielded greater yield stress, higher consistency index, and larger storage and loss moduli, but lower power index. Because more concentrated gels have more stable 3-D structures, they are harder to mobilize (due to their higher yield stress) and harder to deform (due to a higher consistency index and larger storage and loss moduli); they behave more like solids (due to lower power index).

Liu found that the Herschel-Bulkley model was the best fit for the flow curves of the 1X gel [8]. However, her values for yield stress and consistency index were lower and power index was higher than our results; these discrepancies might be the results of wall-slip effects brought on by the smooth surface measuring system. Her values for storage and loss moduli were higher than our results, possibly due to the lower temperature she used in her measurements. She used 20°C for the measuring temperature compared to 41°C in our tests. At lower temperatures, viscosity and flow resistance are higher; thus, storage and loss moduli were greater than those at higher temperatures.

4.1.6 Pressure Gradient without Wall-Slip Effect

Without the effect of wall slip, we calculated the pressure gradient required for gel extrusion through the fracture using yield stress and the simple force balance approach. Assuming the 1X gel was injected into a 1-mm wide fracture, the pressure gradient was 143 kPa/m (6.3 psi/ft). This prediction was comparable to Liu’s—but still far below the results from the extrusion experiments: 633 to 1,130 kPa/m (28 to 50 psi/ft).

The main value of this investigation was that we identified a more reliable means to determine the yield stresses for our 1X, 2X, and 3X gels. We also established that wall
slip was not primarily responsible for the apparent discrepancy of the pressure gradient between the rheology measurements and the extrusion results in fractures.

4.2 FIRST NORMAL STRESS DIFFERENCE ON THE PRESSURE GRADIENT

For the extrusion experiments in 1-mm wide fractures, the injection rates were between 0.046 and 23.4 cm/s, which converted to shear rates (at the wall) between 0.92 and 468 s\(^{-1}\). We performed steady shear measurements in our rheometer to determine the first normal stress difference within this range of shear rates. The pressure gradient that resulted from the first normal stress difference was analyzed and compared with those from the extrusion experiments.

4.2.1 Steady Shear Tests for First Normal Stress Difference

First normal stress difference increased with shear rate for the 1X, 2X, and 3X gels as shown in Figure 4.19. First normal stress difference increased with the polymer and crosslinker concentrations, which is reasonable since more concentrated gels have greater elasticity.

For the 1X gel, the first normal stress difference increased from 442 to 19,000 Pa when the shear rate increased from 0.2 to 1000 s\(^{-1}\). At shear rates below 100 s\(^{-1}\), \(N_f\) was proportional to the square root of shear rate, \(\gamma^{0.5}\). For shear rates between 100 and 1,000 s\(^{-1}\), \(N_f\) was proportional to \(\gamma^{0.25}\).

For the 2X gel, the first normal stress difference increased from 1,000 to 19,900 Pa when the shear rate increased from 0.05 to 100 s\(^{-1}\). When the shear rate was below 0.2 s\(^{-1}\), the slope of the \(N_f\) curve was about 2. At higher shear rates, the first normal stress difference was fairly insensitive to shear rate.
4.19—First normal stress difference for the 1X, 2X, and 3X gels at 41°C.

For the 3X gel, the first normal stress was between 1,000 and 43,200 Pa in the shear rate range from 0.005 to 60 s⁻¹. For shear rates below 0.1 s⁻¹, the slope of the $N_I$ curve was about 1. For shear rates between 0.1 and 100 s⁻¹, the first normal stress difference was fairly insensitive to shear rate. This behavior is qualitatively consistent with polymer behavior that was reported earlier [44].

4.2.2 The Effect of First Normal Stress Difference on the Pressure Gradient

The first normal stress difference depends on shear rate and flow geometry [43]. Therefore, for steady state flow through a fracture with smooth faces, the first normal stress difference should remain constant along the fracture. However, since fracture faces typically have rough surfaces, fracture width typically varies along the fracture length. Thus, even at the same flow rate, the shear rate could be different in different portions of the fracture. As a result, the first normal stress difference could vary along the fracture.
and varying pressures might be exerted on the fracture faces. Can this observation explain the difference in pressure gradient found between the gel extrusion experiments in fractures and our estimation from the rheology measurements?

Assuming that the fracture width was 1 mm when a 1X gel was injected, the predicted pressure gradient was 143 kPa/m (6.3 psi/ft) from the simple force balance approach, compared to 633 to 1,130 kPa/m (28 to 50 psi/ft) in the extrusion experiments [6-7]. Figure 4.19 shows that the first normal stress for the 1X gel was between 442 and 19,000 Pa in the shear rate range from 0.2 to 1000 s⁻¹. If a constriction reduced the effective fracture width by 50%, the shear rate should increase by a factor of 8 (i.e., 2³). From Figure 4.19, this variation in shear rate should not change the normal stress by more than a factor of four for the 1X gel. If normal stress is reduced from 10,000 Pa to 2,500 Pa when passing through a constriction, a pressure change of 7,500 Pa (1.1 psi) might be experienced. If 20 such constrictions were encountered along the fracture, this behavior could explain the higher pressure gradients observed during gel extrusion through fractures. However, during our extrusion experiments, we generally used fractures that did not have obvious constrictions. In reality, we often used fractures with smooth faces, and the pressure gradients during extrusion of a 1X gel through a 1-mm wide fracture were still in the range from 633 to 1,130 kPa/m (28 to 50 psi/ft) [6–7]. Therefore, although normal forces could add significantly to the pressure gradients observed when using fractures with many pronounced constrictions, they do not explain the high pressure gradients observed during the bulk of our gel extrusion experiments.
4.3 GEL DEHYDRATION AND WORMHOLES IN FRACTURES

Before gelation, fluid gelant solutions can readily leak off from fractures into porous rock. However, after gelation, the crosslinked materials will not penetrate significantly into the porous rock [5–7]. Thus, formed gels must extrude through fractures during the placement process. Cr(III)-acetate-HPAM gels (as well as other gels) concentrate or lose water during extrusion through fractures. As water leaks off from the gel, the gel concentrates (by factors up to 50) to become immobile in the vicinity where dehydration occurs. The driving force for gel dehydration (and water leakoff) is the pressure difference between the fracture and the adjacent porous rock. Because fresh gel or mobile gel with the original composition is much more pliable and mobile than the concentrated gel, the fresh gel wormholes through the concentrated gel in order to advance the gel front [5–7]. These wormholes may be significantly narrower than the fracture width. This possibility could explain the relatively high pressure gradients during gel extrusion.

By inputting a pressure gradient of 633 kPa/m (28 psi/ft) and a yield stress of 35 Pa (0.005 psi) (from Liu [8, 18]) into a simple force balance equation, Seright [7] estimated the actual opening width to be about 0.1 mm—one-tenth of the fracture width.

Similarly, using a pressure gradient range of 633 to 1,130 kPa/m (28 to 50 psi/ft) with our measured yield stress of 72 Pa (0.01 psi) for the 1X gel, the calculated apertures ranged from 0.13 to 0.23 mm. Thus, wormhole channels probably were significantly narrower than the fracture width, resulting in much higher pressure gradients during gel extrusion through fractures.
CHAPTER 5. FLOW MODEL OF GELS IN TUBES AND FRACTURES

5.1 SHELL MOMENTUM BALANCE

The simple force balance approach showed that the minimum pressure gradient required for gel extrusion through fractures was inversely proportional to the fracture width, or the capillary radius [19]. However, the extrusion experiments demonstrated that for a given fracture conductivity and width (between 0.15 and 10 mm), the pressure gradient required for gel extrusion through the fracture was inversely proportional to the square of the fracture width [6, 20]. Two flaws existed with the Bingham model [5] when describing the flow behavior of Cr(III)-acetate-HPAM gels in the fracture. The Bingham model’s failure to explain gel behavior in the fracture implied that the Cr(III)-acetate-HPAM gels were not Bingham materials. The simple force balance approach is based on the definition of yield stress: that gels do not flow when shear stress is below the yield stress point; flow begins when shear stress is above the yield stress. The simple force balance approach focused solely on the non-flowing to flowing point. This method, though useful for calculating the minimum pressure gradient required for gel to flow, was not suitable for describing gel flow behavior in fractures.

The rheology measurements described in Chapter 4 showed that the Cr(III)-acetate-HPAM gels were better described by the Herschel-Bulkley model, where the gel exhibited a yield point and flowed as a power-law fluid when shear stress was above the
yield point. Therefore, the shell momentum balance [55] and power-law equations were employed to deduce alternative models for describing gel behavior in fractures.

The shell momentum balance given here applies to steady-state flow, where the flow system at time \( t \) looks exactly the same as at some later time \( t + \Delta t \). For the steady-state flow over a thin shell of fluid, the momentum balance is:

\[
\text{(rate of momentum in)} - \text{(rate of momentum out)} + \text{(sum of forces acting on the system)} = 0
\]

5.2 THE RELATIONSHIP OF PRESSURE GRADIENT VERSUS FRACTURE WIDTH AND TUBE RADIUS

5.2.1 Flow in Tubes

For steady-state flow, a momentum balance for a shell of finite thickness was first applied. As the thickness approached zero, the corresponding differential equation describing the momentum flux distribution was obtained. Next, the appropriate Newtonian or non-Newtonian expression for the momentum flux was inserted to obtain a differential equation for the velocity distribution. Integration of these two differential equations yielded the momentum flux and velocity distributions for the system. This information could then be used to calculate various other quantities, such as average velocity, maximum velocity, volumetric flow rate, pressure drop, and forces on boundaries.

The shell momentum balance for fluid flow in tubes is shown in Figure 5.1. For fluid flows inside a tube

\[
\tau = \frac{P_0 - P_L}{2L} R,
\]

5.1
Figure 5.1—Shell momentum balance for fluid flow in tubes [55].

where $R$ is the radius and $L$ is the length of the tube, and $p_0$ and $p_L$ are the inlet and outlet pressures, respectively.

5.2.1.1 Newton’s Law

For Newtonian material,

$$\tau = -\eta \frac{d\gamma}{dt} = -\eta \frac{dv}{dr},$$  

where $\tau$ is the shear stress, $\eta$ is the Newtonian viscosity, $\gamma$ is shear strain, $t$ is time, $v$ is the velocity, and $r$ is the distance from the tube center.

Combining Eqs. 5-1 and 5-2, the velocity distribution along the radial direction is
\[ v = \frac{P_0 - P_L}{4\pi L} R^2 \left[ \left( \frac{r}{R} \right)^2 - 1 \right] \]  

Integrating Eq. 5-3, the volumetric flow rate is

\[ q = \frac{\pi (P_0 - P_L) R^4}{8\pi L} \]  

The pressure gradient versus tube radius is

\[ \frac{dp}{dl} = \frac{8\pi}{\pi} \frac{q}{R^4} \]  

Thus, for Newtonian materials, the pressure gradient varied inversely with the fourth power of tube radius.

5.2.1.2 Power-Law

For non-Newtonian materials, the power-law is

\[ \tau = -k \left( \frac{dv}{dr} \right)^{n+1} \]

where \( k \) is the consistency index and \( n \) is the power index.

The velocity distribution along the tube radius is

\[ v = \left( -\frac{P_0 - P_L}{2kL} \right)^{\frac{1}{n+1}} \frac{n+1}{n+2} \left( \frac{n+2}{r^{n+1} - R^{n+1}} \right) \]

and the volumetric flow rate is

\[ q = \left( -\frac{P_0 - P_L}{2kL} \right)^{\frac{1}{n+1}} \frac{n+1}{\pi} \left( -\frac{n+1}{3n+4} \right)^{\frac{3n+4}{n+1}} \]

The pressure gradient versus tube radius is

\[ \frac{dp}{dl} = (-2k) \left( -\frac{3n+4}{n+1} \right)^{n+1} \left( \frac{q}{\pi} \right)^{n+1} \frac{1}{R^{3n+4}} \]
Therefore, the pressure gradient varied inversely with the tube radius raised to the power, $3n+4$, for non-Newtonian materials following the power-law.

### 5.2.2 Flow in Fractures

For fluids flowing between two parallel plates separated by a distance of $w$, as shown in Figure 5.2 (for a fracture with a width of $w$), the shell momentum balance reveals that

$$
\tau = \left( \frac{p_0 - p_L}{L} \right) x, \quad 5-10
$$

where $x$ is the distance from the center of the fracture to the fracture wall.

#### 5.2.2.1 Newton's Law

Following the same procedures used for tube flow, for Newtonian materials the velocity distribution along the fracture width ($w$) is

$$
v = \frac{p_0 - p_L}{2\eta L} w^2 \left[ \left( \frac{x}{w} \right)^2 - \frac{1}{4} \right], \quad 5-11
$$

The volumetric flow rate is

$$
q = \frac{1}{12} \frac{(p_0 - p_L)}{\eta L} hw^3, \quad 5-12
$$

where $h$ is the fracture height and $L$ is the fracture length.
Figure 5.2—Shell momentum balance for fluid flowing in the fracture.

The pressure gradient versus fracture width is

\[
\frac{dp}{dl} = \frac{12\eta q}{h} \frac{1}{w^3}
\] 5-13

Therefore, the pressure gradient varied inversely with the third power of fracture width for Newtonian materials.

5.2.2.2 Power-Law

For non-Newtonian materials following the power-law, the velocity distribution along the fracture width is

\[
v = \left( -\frac{p_0 - p_k}{kL} \right)^{\frac{1}{n+1}} \frac{n+1}{n+2} \left( \frac{w}{2} \right)^{\frac{n+2}{n+1}} \left[ \left( \frac{2x}{w} \right)^{\frac{n+2}{n+1}} - 1 \right],
\] 5-14
and the volumetric flow rate is

\[ q = \left( -\frac{P_0 - P_L}{2kL} \right)^{\frac{1}{n+1}} h \left( -\frac{n+1}{2n+3} \right)^{\frac{2n+3}{n+1}} \]

The pressure gradient versus fracture width is

\[ \frac{dp}{dl} = (-2k) \left( -\frac{2n+3}{n+1} \right)^{n+1} \left( \frac{2q}{h} \right)^{n+1} \frac{1}{w^{2n+3}} \]

Therefore, for non-Newtonian materials following the power-law, the pressure gradient varied inversely with the fracture width, raised to the power, \(2n+3\).

5.2.3 Pressure Gradient versus Tube Radius and Fracture Width

Summarizing the above equations, the pressure gradients required for Newtonian materials to extrude varied inversely with the fourth power of tube radius or with the third power of fracture width. For non-Newtonian materials following the power-law, the pressure gradients varied inversely with tube radius raised to the power, \(3n+4\), or with fracture width raised to the power, \(2n+3\).

The relationship of pressure gradient versus tube radius or fracture width for our 1X, 2X, and 3X gels are summarized in Table 5.1. For the equations listed in this table, \(q\) must have units of \(m^3/s\); \(R\), \(w\), and \(h\) must have units of \(m\); \(\eta\) must have units of \(Pa\cdot s\); and the calculated pressure gradients, \(dp/dl\), have units of \(Pa/m\).

During extrusion experiments, Seright [5-7] noted that the pressure gradient varied inversely with the square of the fracture width. In examining the gel entries for Table 5.1, the exponents ranged from 1.57 to 1.78 for the \(R\) variables and from 1.38 to 1.52 for the \(w\) variables. These exponents were intermediate between the exponent of one
(predicted from a simple force balance with Newtonian fluids) and two (the approximate value observed experimentally).

**Table 5.1—Pressure gradient versus aperture for the Cr(III)-acetate-HPAM gels.**

<table>
<thead>
<tr>
<th>Materials</th>
<th>Tube</th>
<th>Fracture</th>
</tr>
</thead>
<tbody>
<tr>
<td>Newtonian</td>
<td>(\frac{dp}{dl} = 8\eta \frac{q}{\pi R^4})</td>
<td>(\frac{dp}{dl} = \frac{12\eta q}{h} \frac{1}{w^3})</td>
</tr>
<tr>
<td>1X gel</td>
<td>(\frac{dp}{dl} = 101q^{0.26} \frac{1}{R^{1.78}})</td>
<td>(\frac{dp}{dl} = 156\left(\frac{q}{h}\right)^{0.26} \frac{1}{w^{1.52}})</td>
</tr>
<tr>
<td>2X gel</td>
<td>(\frac{dp}{dl} = 532q^{0.21} \frac{1}{R^{1.63}})</td>
<td>(\frac{dp}{dl} = 760\left(\frac{q}{h}\right)^{0.21} \frac{1}{w^{1.42}})</td>
</tr>
<tr>
<td>3X gel</td>
<td>(\frac{dp}{dl} = 1300q^{0.19} \frac{1}{R^{1.57}})</td>
<td>(\frac{dp}{dl} = 1796\left(\frac{q}{h}\right)^{0.19} \frac{1}{w^{1.38}})</td>
</tr>
</tbody>
</table>

5.2.3.1 Tube Flow

Figure 5.3 shows plotted values for pressure gradient versus tube ID (inside diameter) for the 1X gel at a fixed superficial velocity of 10,700 m/d. The pressure gradients calculated by the power-law model were higher than those calculated by the simple force balance approach. For tubes with inside diameter (ID) between 0.8 and 2 mm, the power-law model's calculated pressure gradients were a closer match to the extrusion data than those calculated by the simple force balance approach. However, when tube ID was less than 0.8 mm and greater than 2 mm, the power-law model's calculated pressure gradients were higher than the experimentally observed values. More work is needed to understand this discrepancy.
Figure 5.3—Comparison of flow models with extrusion experiment for 1X gel in tubes.

Figure 5.4—Comparison of flow models with extrusion experiment for 1X gel in fractures.
Therefore, the power-law model described the relation between the pressure gradient and the tube ID fairly well, as long as the tube ID was neither too small (e.g., ID was not less than 0.8 mm) nor too large (e.g., ID was not greater than 2 mm).

5.2.3.2 Fracture Flow

Figure 5.4 shows plotted values for pressure gradient versus fracture width for the 1X gel at a fixed volumetric rate of 16,000 ml/hr. Over the experimental range, the pressure gradients estimated by the power-law model were greater than those calculated by the simple force balance approach. For fracture widths less than 2.54 mm, the power-law model's calculated pressure gradients were a closer match to the extrusion data than those calculated by the simple force balance approach. However, for wider fractures (e.g., for fracture widths greater than 3 mm), the power-law model overestimated the pressure gradients required for the gel to flow. More work is needed to understand this discrepancy.

Therefore, the power-law model described the relation between the pressure gradient and the fracture width fairly well, as long as the fracture was not too wide (e.g., the fracture width was not greater than 2.54 mm).

5.2.4 Velocity Distribution versus Fracture Width and Tube Radius

The velocity distribution versus aperture width was calculated for the 1X, 2X, and 3X gels flowing through a 38.1-mm high by 1-mm wide fracture, and for a tube with 2 mm ID with a flow rate of 16,000 ml/hr. The flow curves in Figures 5.5 and 5.6 indicate that the gels moved more like a piston through the apertures than did the Newtonian fluid, demonstrating the effective combination of shell momentum balance and gel rheology properties to analyze flow behaviors when gels are extruded through fractures.
Figure 5.5—Velocity distribution for the 1X, 2X, and 3X gels in tubes.

Figure 5.6—Velocity distribution for the 1X, 2X, and 3X gels in fractures.
5.3 SIMILAR RELATIONSHIPS BETWEEN PRESSURE GRADIENT VERSUS FLOW RATE AND SHEAR STRESS VERSUS SHEAR RATE

The behavior of shear stress versus shear rate and pressure gradient versus superficial velocity might be related to the gel behavior in a fracture versus in a rheometer during extrusion. If it is possible to correlate these variables, the reason for this phenomenon might be discovered. To develop these relationships, we employed a shell momentum balance and power-law equations. Pressure gradients were correlated with shear stresses using Equation 5-1 for fluid flow in tubes, and using Equation 5-10 for fluid flow in fractures.

Combining Equations 5-1, 5-2 and 5-4 relates the shear rate to the flow rate for Newtonian materials in tubes:

\[ \dot{\gamma} = \frac{4q}{\pi R^4} r \]  

Combining Equations 5-1, 5-7, and 5-9 gives the shear rate in tubes for non-Newtonian materials following the power-law:

\[ \dot{\gamma} = \frac{3n+4}{n+1} \frac{q}{\pi R} \frac{1}{x^{n+1}} \]  

Combining Equations 5-2, 5-11, and 5-12 gives the shear rate for Newtonian materials in fractures:

\[ \dot{\gamma} = \frac{12q}{hw^4} x \]  

Combining Equations 5-7, 5-11, and 5-16 gives the shear rate in fractures for non-Newtonian materials following power-law:

\[ \dot{\gamma} = \frac{2n+3}{n+1} \frac{2q}{h} \frac{1}{w^{n+1}} (2x)^{n+1} \]
Table 5.2 shows the correlations of pressure gradient versus shear stress and flow rate versus shear rate at the wall, where \( r=R \) or \( x=w/2 \), for the 1X, 2X, and 3X gels.

**Table 5.2—Correlation of pressure gradient versus shear stress and flow rate versus shear rate at the wall.**

<table>
<thead>
<tr>
<th>Materials</th>
<th>Tube</th>
<th>Fracture</th>
</tr>
</thead>
<tbody>
<tr>
<td>1X gel n=0.745</td>
<td>( \dot{\gamma} = -6.93 \frac{q}{\pi R^3}, \tau = \frac{p_0 - p_L}{L} )</td>
<td>( \dot{\gamma} = -11.86 \frac{q}{hw^2}, \tau = \frac{p_0 - p_L}{L} )</td>
</tr>
<tr>
<td>2X gel n=0.787</td>
<td>( \dot{\gamma} = -7.69 \frac{q}{\pi R^3}, \tau = \frac{p_0 - p_L}{L} )</td>
<td>( \dot{\gamma} = -13.38 \frac{q}{hw^2}, \tau = \frac{p_0 - p_L}{L} )</td>
</tr>
<tr>
<td>3X gel n=0.806</td>
<td>( \dot{\gamma} = -8.14 \frac{q}{\pi R^3}, \tau = \frac{p_0 - p_L}{L} )</td>
<td>( \dot{\gamma} = -14.28 \frac{q}{hw^2}, \tau = \frac{p_0 - p_L}{L} )</td>
</tr>
</tbody>
</table>

### 5.3.1 Comparison with Tube Flow

Using the correlations in Table 5.2, we calculated the shear stress and shear rate values for the 1X gel from the pressure gradients and flow rates in the extrusion experiments. The calculated values were compared to the rheology measurements made with a gap height of 1 mm on smooth and rough surfaces. For tube flow (Figure 5.7), the calculated shear-stress versus shear-rate relation followed the same trend as the rheology measurements for shear rates above 100 s\(^{-1}\). However, the measured shear stresses were higher than the calculated values for tube flow when shear rates were below 100 s\(^{-1}\).
Figure 5.7—Comparison of flow curves in tubes for 1X gel.

Figure 5.8—Comparison of flow curves in fractures for 1X gel.
5.3.2 Comparison with Fracture Flow

For a 2-mm wide fracture (Figure 5.8), the calculated shear-stress versus shear-rate relation followed the same trend as the rheology measurements. However, for narrower fractures (0.254 mm and 1 mm), the calculated shear stress values from the extrusion experiments were generally much higher than the rheology measurements. Since the highest shear stress values were noted in the narrowest fracture, this discrepancy might be explained by the fairly rough Berea sandstone surface seriously interfering with gel flow in the fracture, especially for very narrow fractures. For wider fractures (e.g., 2 mm) this interference became negligible; thus, the flow curve was similar to our rheology measurements.

The similarity between the flow curves from the fracture flow or tube flow and the rheology measurements helps to bridge the gap between gel behaviors in fractures or tubes with those seen in a rheometer. However, additional work is needed to fully explain the high pressure gradients observed during gel extrusion through narrow fractures and the low pressure gradient in tube flow for shear rates below 100 s$^{-1}$. 
CHAPTER 6. CONCLUSIONS AND FUTURE WORK

6.1 CONCLUSIONS

When evaluating the use of gels for conformance improvement in treating channeling through fractures, rheology measurements can be made much faster and at much lower cost than measurements made during extrusion of gels through fractured cores. However, when using the simple force balance approach and the rheology measurements to predict gel behavior in fractures, we encountered two apparent discrepancies. This research examined both problems with the hope of ultimately substituting rheology measurements for extrusion experiments. The following conclusions were reached, based on studies of the Cr(III)-acetate-HPAM gels at 41°C.

1. Wall-slip effects existed and reduced the measured value of yield stress on the smooth surface measuring system. Use of double-rough, parallel plates in our rheometer significantly reduced the importance of wall slip (compared to smooth plates). However, the higher stresses noted during the absence of slip effects were not enough to explain the high pressure gradients during gel extrusion experiments.

2. Consistent with our expectations, the first normal stress difference increased with shear rate and polymer concentration for our Cr(III)-acetate-HPAM gels. Although first normal stress might result in greater pressure gradients during extrusion through fractures with significant constrictions, it was not great enough to explain the high pressure gradients during extrusion experiments.
3. Earlier work revealed that gels propagate through fractures by wormholing through immobile concentrated (dehydrated) gel. Since these wormholes are narrower than the fracture width, this could partly explain the higher pressure gradients during gel extrusion experiments.

4. A model based on power-law equations and shell momentum balance predicted that pressure gradient should vary with fracture width and tube radius raised to a power between $-1$ and $-2$. This model fit our extrusion data reasonably well for fracture widths of 2 mm or less and tube ID between 0.8 and 2 mm.

5. The power-law model did well in correlating pressure gradient with shear stress and flow rate with shear rate in 2-mm wide fractures and for tube flow when shear rate was above 100 s$^{-1}$. However, this model was notably less successful in narrower fractures and in tube flow when the shear rate was below 100 s$^{-1}$. Additional work is needed to fully explain the high pressure gradients observed during gel extrusion through narrow fractures and the low pressure gradients in tube flow when shear rate was below 100 s$^{-1}$.

6.2 FUTURE WORK

Based on the experimental results and conclusions drawn from this work, a few questions remain that could be the focus for future work.

- What is the flow channel size during extrusion?

- What can be done to fix the problems when using the power-law model to predict gel behavior in fractures (e.g., less successful in describing gel behavior in narrower and wider fractures and tube and for lower shear rate in tube flow)?
Can the power-law model, based on the shell momentum balance and power-law equations, be used to predict flow behaviors of other polymers and gels during extrusion through fractures and tubes?
REFERENCES


APPENDIX

A.1 SHELL MOMENTUM BALANCE

A.1.1 Shell Momentum Balance in a Tube

For the steady-state flow over a thin shell of fluid, the momentum balance is

\[ \text{rate of momentum in} - \text{rate of momentum out} + \text{sum of forces acting on the system} = 0 \]

To make the shell momentum balance in a tube, the following conditions must be satisfied: steady-state flow (where the flow system at time \( t \) looks exactly the same as at some later time \( t + \Delta t \)), constant density \( \rho \) of the fluid, and no "end effect" (e.g., in a "very long" tube of length \( L \) and radius \( R \)).

Consider Figure A1, which shows a cylindrical shell of thickness \( \Delta r \) and length \( L \), and first list the various contributions to the momentum balance in the z-direction:

Rate of momentum "in" across the cylindrical surface at \( r \) is

\[ (2\pi r L \tau_r) \]

Rate of momentum "out" across the cylindrical surface at \( r + \Delta r \) is

\[ (2\pi (r + \Delta r) L \tau_r) \]

Rate of momentum "in" across the annular surface at \( z = 0 \) is

\[ (2\pi \Delta r v_z)(\rho v_z)_{z=0} \]

\[ a-1 \]

\[ a-2 \]

\[ a-3 \]
Rate of momentum "out" across the annular surface at $z = L$ is

$$(2\pi r \Delta r v_z)(\rho v_z)_{z=L}$$  \hspace{1cm} \text{(a-4)}$$

Gravity force acting on the cylindrical shell is

$$(2\pi r \Delta r L) \rho g$$  \hspace{1cm} \text{(a-5)}$$

Pressure force acting on the annular surface at $z = 0$ is

$$(2\pi r \Delta r) p_0$$  \hspace{1cm} \text{(a-6)}$$

Pressure force acting on the annular surface at $z = L$ is

$$-(2\pi r \Delta r) p_L$$  \hspace{1cm} \text{(a-7)}$$

Here, "in" and "out" are the positive directions of the axes.
Next, add all the contributions to the momentum balance:

\[ \left( 2\pi rL \tau_r \right)_r - \left( 2\pi rL \tau_r \right)_{r+\Delta r} + \left( 2\pi r\Delta r \rho v_z^2 \right)_{z=0} - \left( 2\pi r\Delta r \rho v_z^2 \right)_{z=L} + 2\pi r\Delta rL \rho g + 2\pi r\Delta r (p_0 - p_L) = 0 \]

Assuming that the fluid is incompressible, \( v_z \) is the same at \( z = 0 \), and \( z = L \); hence the third and fourth terms cancel one another as follows:

\[ (2\pi rL \tau_r)_r - (2\pi rL \tau_r)_{r+\Delta r} + 2\pi r\Delta rL \rho g + 2\pi r\Delta r (p_0 - p_L) = 0 \]

Dividing Equation a-9 by \( 2\pi rL \Delta r \) results in

\[ \frac{[(r \tau_r)_r - (r \tau_r)_{r+\Delta r}] / \Delta r + r \rho g + r (p_0 - p_L) / L = 0} \]

Taking the limit as \( \Delta r \) goes to zero gives

\[ \lim_{\Delta r \to 0} \frac{(r \tau_r)_{r+\Delta r} - (r \tau_r)_r}{\Delta r} = \frac{p_0 - p_L}{L} + \rho g r \]

The left side is the definition for the first derivative:

\[ \lim_{\Delta r \to 0} \frac{(r \tau_r)_{r+\Delta r} - (r \tau_r)_r}{\Delta r} = \frac{d}{dr} (r \tau_r) \]

Taking \( P_0 = p_0 - \rho g \), \( P_L = p_L - \rho gL \) gives

\[ \frac{d}{dr} (r \tau_r) = \left( \frac{P_0 - P_L}{L} \right) r \]

Integrating Equation a-13 gives

\[ \tau_r = \left( \frac{P_0 - P_L}{2L} \right) r + \frac{C_1}{r} \]

\( C_1 \) is zero because the momentum cannot be infinite at \( r = 0 \); for tube flow, shear stress is represented by the pressure gradient as

\[ \tau_r = \left( \frac{P_0 - P_L}{2L} \right) r \]
A.1.2 Shell Momentum Balance in a Fracture

Figure A2—Fluid flow in a slit.

A fluid is in laminar flow in a slit formed by two parallel walls a distance of \( w \) apart, in a shell of thickness \( \Delta x \) and length \( L \) (Figure A2). Start by listing the various contributions to the momentum balance in the \( z \)-direction:

Rate of momentum "in" surface at \( x \) is

\[
(hL\tau_{xz})_x
\]

(a-15)

Rate of momentum "out" surface at \( x + \Delta x \) is

\[
(hL\tau_{xz})_{x+\Delta x}
\]

(a-16)
Rate of momentum “in” surface at \( z = 0 \) is

\[
(h\Delta x \nu_z)(\rho \nu_z)_{z=0}
\]

Rate of momentum “out” surface at \( z = L \) is

\[
(h\Delta x \nu_z)(\rho \nu_z)_{z=L}
\]

Gravity force acting on the shell is

\[
(h\Delta x L) \rho g
\]

Pressure force acting on the surface at \( z = 0 \) is

\[
(h\Delta x) p_0
\]

Pressure force acting on the surface at \( z = L \) is

\[
-(h\Delta x) p_L
\]

Here, “in” and “out” are the positive directions of the axes.

Next, add all the contributions to the momentum balance:

\[
(hL \tau_{xz})_x - (hL \tau_{xz})_{x+\Delta x} + (h\Delta x \rho \nu_z^2)_{z=0}
- (h\Delta x \rho \nu_z^2)_{z=L} + h\Delta x L \rho g + h\Delta x (p_0 - p_L) = 0
\]

Assuming that the fluid is incompressible, \( \nu_z \) is the same at \( z = 0 \), and \( z = L \); hence, the third and fourth terms cancel one another as follows:

\[
(hL \tau_{xz})_x - (hL \tau_{xz})_{x+\Delta x} + h\Delta x L \rho g + h\Delta x (p_0 - p_L) = 0
\]

Dividing Equation a-23 by \( hL \Delta x \) gives

\[
[(\tau_{xz})_x - (\tau_{xz})_{x+\Delta x}] / \Delta x + \rho g + (p_0 - p_L) / L = 0
\]

Taking the limit as \( \Delta x \) goes to zero and results in

\[
\lim_{\Delta x \to 0} \frac{(\tau_{xz})_{x+\Delta x} - (\tau_{xz})_x}{\Delta x} = \frac{p_0 - p_L}{L} + \rho g
\]
The left side is the definition for the first derivative:

\[
\lim_{\Delta x \to 0} \frac{(\tau_x)_{x+\Delta x} - (\tau_x)_x}{\Delta x} = \frac{d\tau_x}{dx}
\]

Taking \( P_0 = p_0 - \rho g \), \( P_L = p_L - \rho g L \) gives

\[
\frac{d\tau_x}{dx} = \frac{P_0 - P_L}{L}
\]

Integrating Equation a-27 results in

\[
\tau_x = \left( \frac{P_0 - P_L}{L} \right)x + C_2
\]

C_2 is zero because the shear stress cannot be infinite at \( x = 0 \); for fracture flow, shear stress is represented by the pressure gradient as

\[
\tau_x = \left( \frac{P_0 - P_L}{L} \right)x
\]

A.2 NEWTONIAN LIQUID FLOWING IN TUBES

The definition for shear rate in cylindrical coordinates is expressed by

\[
\dot{\gamma} = \frac{dv_z}{dr}
\]

Newton’s Law of viscosity is

\[
\tau_x = -\eta \dot{\gamma} = -\eta \frac{dv_z}{dr}
\]

Substitution of Equation a-30 into Equation 5-1 results in

\[
\frac{dv_z}{dr} = -\frac{P_0 - P_L}{2\eta L} r
\]
Integrating Equation a-31 gives

\[ v_z = -\frac{P_0 - P_L}{4\eta L} r^2 + C_3 \]  

a-32

Because of the boundary condition that \( v_z \) be zero at \( r = R \),

\[ C_3 = \frac{P_0 - P_L}{4\eta L} R^2 \]  

a-33

Therefore, the velocity distribution along the radius direction is

\[ v_z = -\frac{P_0 - P_L}{4\eta L} R^2 \left[ \left( \frac{r}{R} \right)^2 - 1 \right] \]  

5-3

Integrating Equation 5-3, gives the volumetric flow rate—Hagen-Poiseuille Law:

\[ q = \int_0^{2\pi} \int_0^R v_z r dr d\theta \]

\[ q = \int_0^{2\pi} \int_0^R -\frac{P_0 - P_L}{4\eta L} R^2 \left[ \left( \frac{r}{R} \right)^2 - 1 \right] r dr d\theta \]

\[ q = \frac{\pi (P_0 - P_L)}{8\eta L} R^4 \]  

5-4

Rearranging Equation 5-4, the pressure gradient versus tube radius is represented by

\[ \frac{dp}{dl} = \frac{P_0 - P_L}{L} = \frac{8\eta q}{\pi R^4} \]  

5-5

Rearranging Equation 5-3 gives

\[ \frac{P_0 - P_L}{L} = \frac{4\eta v_z}{-r^2 + R^2} \]  

a-34

Substitution of Equation a-34 into Equation 5-5 gives

\[ v_z = -\frac{2q}{\pi R^2} \left[ \left( \frac{r}{R} \right)^2 - 1 \right] \]  

a-35
Substitution of Equation a-35 into Equation a-29, the shear rate was correlated with the volumetric flow rate as follows:

\[
\dot{\gamma} = \frac{-2q}{\pi R^2} \left[ \left( \frac{r}{R} \right)^2 - 1 \right] \frac{d}{dr}
\]

\[
\dot{\gamma} = -\frac{4q}{\pi R^4} r
\]

5-17

Shear rate at the wall, \( r = R \), is

\[
\dot{\gamma}_w = \frac{4q}{\pi R^3}
\]

a-36

A.3 NON-NEWTONIAN MATERIALS FLOWING IN TUBE

The power-law equation for Non-Newtonian materials is

\[
\tau_{xz} = -k \dot{\gamma} = -k \left( \frac{dv_z}{dr} \right)^{n+1}
\]

a-37

Substitution of Equation a-37 into Equation 5-1 gives

\[
\frac{dv_z}{dr} = \left( -\frac{P_0 - P_L}{2kL} \right) \frac{1}{n+1} \frac{1}{r \frac{n+1}{n+1}}
\]

a-38

Integrating Equation a-38 gives

\[
v_z = \left( -\frac{P_0 - P_L}{4kL} \right) \frac{1}{n+1} \frac{n+1}{n+2} \frac{n+2}{r \frac{n+1}{n+1}} + C_4
\]

a-39

Because of the boundary condition that \( v_z \) be zero at \( r = R \),

\[
C_4 = \left( -\frac{P_0 - P_L}{4kL} \right) \frac{1}{n+1} \frac{n+1}{n+2} \frac{n+2}{R \frac{n+1}{n+1}}
\]

a-40
Therefore, the velocity distribution along the radius direction is

$$v_z = \left( -\frac{P_0 - P_L}{4kL} \right)^{n+1} \frac{1}{n+2} \left[ \frac{\frac{n+2}{r^{n+1}}}{R^{n+1}} - \frac{\frac{n+2}{r^{n+1}}}{R^{n+1}} \right]$$ 5-7

Integrating Equation 5-7, we get the volumetric flow rate:

$$q = \int_0^{\theta} \int_0^{\phi} v_z r dr d\phi$$

$$q = \int_0^{\theta} \int_0^{\phi} \left( -\frac{P_0 - P_L}{4kL} \right)^{n+1} \frac{1}{n+2} \left[ \frac{\frac{n+2}{r^{n+1}}}{R^{n+1}} - \frac{\frac{n+2}{r^{n+1}}}{R^{n+1}} \right] r dr d\phi$$

$$q = \left( -\frac{P_0 - P_L}{2kL} \right)^{n+1} \frac{1}{n+2} \left[ \left( -\frac{n+1}{3n+4} \right) \frac{1}{R^{n+1}} \right]$$ 5-8

Rearranging Equation 5-8 gives the pressure gradient versus tube radius:

$$\frac{dp}{dl} = \frac{P_0 - P_L}{L} = (-2k) \left[ \frac{3n+4}{n+1} \right] \left( \frac{q}{\pi} \right)^{n+1} \frac{1}{R^{3n+4}}$$ 5-9

Substitution of Equation 5-9 into Equation 5-7 gives

$$v_z = -\frac{3n+4}{n+2} q \frac{r^{n+2}}{\pi R^2} \left[ \left( \frac{r}{R} \right)^{n+1} - 1 \right]$$ a-41

Substitution of Equation a-41 into Equation a-29, correlates the shear rate with the volumetric flow rate:

$$\dot{\gamma} = \frac{\frac{3n+4}{n+2} q}{\pi R^2} \left[ \left( \frac{r}{R} \right)^{n+1} - 1 \right]$$

$$\dot{\gamma} = -\frac{3n+4}{n+1} q R \frac{3n+4}{n+1} \frac{1}{r^{n+1}}$$ 5-18
Shear rate at the wall, \( r = R \), is

\[
\dot{\gamma}_w = -\frac{3n + 4}{n + 1} \frac{q}{\pi R^3}
\]

A.4 NEWTONIAN FLUID FLOWING IN FRACTURES

The definition of shear rate in fractures is

\[
\dot{\gamma} = \frac{dv_z}{dx}
\]

Newton's Law of viscosity is

\[
\tau_x = -\eta \dot{\gamma} = -\eta \frac{dv_z}{dx}
\]

Substitution of Equation a-44 into Equation 5-10 gives

\[
\frac{dv_z}{dx} = -\frac{P_0 - P_L}{\eta L} x
\]

Integrating Equation a-45 gives

\[
v_z = -\frac{P_0 - P_L}{2\eta L} x^2 + C_5
\]

Because of the boundary condition that \( v_z \) be zero at \( x = w/2 \),

\[
C_5 = \frac{P_0 - P_L}{8\eta L} w^2
\]

Therefore, the velocity distribution is

\[
v_z = -\frac{P_0 - P_L}{2\eta L} w^2 \left[ \frac{x^2}{w^2} - \frac{1}{4} \right]
\]
Integrating Equation 5-11 we get the volumetric flow rate:

\[ q = 2 \int_0^{w/2} v_z \, dx \, dy \]

\[ q = 2 \int_0^{w/2} \left( \frac{P_0 - P_L}{2 \eta L} \right) w^2 \left[ \left( \frac{x}{w} \right)^2 - \frac{1}{4} \right] \, dx \, dy \]

\[ q = \frac{P_0 - P_L}{12 \eta L} hw^3 \tag{5-12} \]

Rearranging Equation 5-12 gives the pressure gradient versus tube radius:

\[ \frac{dp}{dl} = \frac{P_0 - P_L}{L} = \frac{12 \eta q}{h \, w^3} \tag{5-13} \]

Substitution of Equation 5-13 into Equation 5-11 gives

\[ v_z = -\frac{6q}{hw} \left[ \left( \frac{x}{w} \right)^2 - \frac{1}{4} \right] \tag{a-48} \]

Substitution of Equation a-48 into Equation a-29, the shear rate was correlated with the volumetric flow rate:

\[ \dot{\gamma} = \frac{d \left\{ -\frac{6q}{hw} \left[ \left( \frac{x}{w} \right)^2 - \frac{1}{4} \right] \right\}}{dx} \]

\[ \dot{\gamma} = -\frac{12q}{hw^3} \, x \tag{5-19} \]

Shear rate at the wall, \( x = w/2 \), is

\[ \dot{\gamma}_w = -\frac{6q}{hw^2} \tag{a-49} \]
A.5 NON-NEWTONIAN MATERIALS FLOWING IN FRACTURES

Power-Law for Non-Newtonian materials:

\[ \tau_{xy} = -k \dot{\gamma} = -k \left( \frac{dv_z}{dx} \right)^{n+1} \]

Substitution of Equation a-50 into Equation 5-10 gives

\[ \frac{dv_z}{dr} = \left( -\frac{P_0 - P_L}{kL} \right)^{n+1} \frac{1}{x^{n+1}} \]

Integrating Equation a-51 gives

\[ v_z = \left( -\frac{P_0 - P_L}{kL} \right)^{n+1} \frac{n+1}{n+2} \frac{x^{n+2}}{n+1} + C_6 \]

Because of the boundary condition that \( v_z \) be zero at \( x = w/2 \),

\[ C_6 = -\left( -\frac{P_0 - P_L}{kL} \right)^{n+1} \frac{n+1}{n+2} \left( \frac{2x}{w} \right)^{n+2} \]

Therefore, the velocity distribution is

\[ v = \left( -\frac{P_0 - P_L}{kL} \right)^{n+1} \frac{n+1}{n+2} \left( \frac{w}{n+1} \right)^{n+1} \left( \left( \frac{2x}{w} \right)^{n+2} - 1 \right) \]

Integrating Equation 5-14 gives the volumetric flow rate:

\[ q = 2 \int_0^{w/2} \int_0^{w/2} v_z dx dy \]

\[ q = 2 \int_0^{w/2} \int_0^{w/2} \left( -\frac{P_0 - P_L}{kL} \right)^{n+1} \frac{n+1}{n+2} \left( \frac{w}{n+1} \right)^{n+1} \left( \left( \frac{2x}{w} \right)^{n+2} - 1 \right) dx dy \]

\[ q = \left( -\frac{P_0 - P_L}{2kL} \right)^{n+1} \frac{h}{2} \left( -\frac{n+1}{2n+3} \right)^{2n+3} \frac{w^{n+1}}{n+1} \]

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Rearranging Equation 5-15 gives the pressure gradient versus tube radius:

\[
\frac{dp}{dl} = (-2k) \left( -\frac{2n+3}{n+1} \right)^{n+1} \left( \frac{2q}{h} \right)^{n+1} \frac{1}{w^{2n+3}}
\]

Substitution of Equation 5-16 into Equation 5-14 gives

\[
\nu_\tau = \frac{2n+3}{n+2} \frac{q}{hw} \left[ \left( \frac{2x}{w} \right)^{n+1} - 1 \right]
\]

Substitution of Equation a-54 into Equation a-29, the shear rate was correlated with the volumetric flow rate as

\[
\dot{\gamma} = \frac{d\left\{ -\frac{2n+3}{n+1} \frac{q}{hw} \left[ \left( \frac{2x}{w} \right)^{n+1} - 1 \right] \right\}}{dx}
\]

\[
\dot{\gamma} = -\frac{2n+3}{n+1} \frac{q}{2h} \left( \frac{w}{2} \right)^{2n+3} \frac{1}{x^{n+1}}
\]

Shear rate at the wall, \( x = w/2 \), is

\[
\dot{\gamma}_w = -\frac{2n+3}{n+1} \frac{q}{2} \frac{2}{hw^2}
\]