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Placement of Gels To Modify Injection Profiles

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ABSTRACT

This study investigates how flow profiles in injection wells are modified when zones are not isolated during placement of gelling agents. Mathematical models are used to examine the degree of gel penetration and injectivity loss in zones of different permeability. Several conclusions are drawn that apply to reservoirs in which crossflow between layers does not occur. First, zone isolation is far more likely to be needed during placement of gels in unfractured wells than in fractured wells. Productive zones in unfractured wells may be seriously damaged if zones are not isolated during gel placement. Second, gel placement without zone isolation should cause the least damage to productive zones in unfractured wells when (a) the gelling formulation exhibits a low resistance factor during placement, (b) the water-oil mobility ratio is relatively high, (c) the most-permeable layer(s) are watered-out, and (d) the waterfronts are not close to the production well in the productive zones. Third, parallel linear corefloods overestimate the degree of profile modification that can be attained in radial systems. Fourth, chemical retention, dispersion and diffusion will probably not significantly mitigate injectivity losses caused by gel penetration into low-permeability zones. Finally, a need exists to determine the permeability and velocity dependencies of gelling-agent resistance factors and of gel residual resistance factors.

INTRODUCTION

In the period between the implementation of the Windfall Profits Tax Act of 1980 and the collapse of oil prices in 1986, treatment of injection wells with gels soared in popularity.[1] These treatments have been labelled "profile modification," "profile control," "conformance control," "fluid diversion" and "crosslinked polymer," as well as trade names. The objective of these treatments is to block fractures or watered-out, high-permeability zones such that fluids injected subsequently are more likely to enter and displace oil from other strata.

In most cases when gelling agents are injected to alter flow profiles in a well, zone isolation techniques are not used, so the gelling agent has access to all open intervals. Much of the gelling agent will enter fractures and/or high-permeability zones. However, some of this fluid will penetrate into strata that one does not want to plug. This study uses mathematical models to address several important questions that arise when this happens:

1. If gelling agents are allowed to enter productive zones, how far will they penetrate?
2. How much will the gel impair fluid injectivity in productive zones?
3. How much can injection profiles be expected to change if gels are allowed to enter all zones?
4. If gels are prevented from entering oil-productive zones, how much improvement in injection profile can be expected?

The analysis presented here considers the fortuitous case in which reservoir layers are separated by impermeable barriers. Previous research[2,3] has demonstrated that near-wellbore treatments are not likely to be effective if extensive crossflow between layers can occur. Crossflow allows injected fluids to circumvent small or moderate-sized plugs placed in the high-permeability zones.

The gelling fluids considered here are capable of penetrating fairly deeply (fifty feet or more) into porous rock matrixes. These fluids can be either Newtonian, such as solutions of monomers that may be polymerized to form gels,[4-6] or non-Newtonian,

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References and illustrations at end of paper.

such as solutions of polymers that may be cross-linked.[7-9] Suspensions of particulates, such as ground walnut shells, oyster shells, mothballs or organic resins[10] are not considered here. The behavior of the latter "diverting agents" have been modeled previously.[11,12]

UNFRACTURED SYSTEMS

Degree of Penetration of Gelling Agents. The first objective of this analysis is to establish how deeply gelling agents will penetrate into unfractured zones with different permeabilities during unrestricted injection.

Linear vs. Radial Corefloods with Newtonian Fluids. To begin the analysis, consider the simple case of injecting a Newtonian fluid (e.g., a solution of monomers prior to gelation) for miscible displacement of water from a number of parallel linear cores that have the same length and that share a common injection port. The cores may have different permeabilities, but each is homogeneous and contains no mobile oil. (Laboratory arrangements of this kind have been used to argue the benefits of certain gel systems.[6,9]) For simplicity it is also assumed that the displacement is piston-like, that fluids are incompressible, that no adsorption or dispersion occurs and that gelation is slow relative to the placement process. When the injected fluid reaches the outlet (L_t) of the most-permeable core (core 1), the degree of penetration (L_{pi}/L_t) into a less-permeable core (core i) will be

$$\frac{L_{pi}}{L_t} = \frac{\left[1 + (F_r^2-1)\left(\frac{\phi_1 k_i}{\phi_i k_1}\right)\right]^{0.5} - 1}{F_r-1} \qquad (1)$$

Here, resistance factor (F_r) is assumed to be independent of permeability (k). The above assumptions will probably be valid for monomeric gelling agents, but some may not be valid for gelling agents that contain polyacrylamide or xanthan. The impact of adsorption, dispersion, inaccessible pore volume, permeability-dependent resistance factors and other factors will be discussed later.

Of course, the flow geometry surrounding an unfractured injection well is radial rather than linear. The expression that is analogous to L_{pi}/L_t that describes the degree of penetration during parallel radial corefloods is $(r_{pi}-r_o)/(r_e-r_o)$, where r_{pi} is the radius of the injected fluid in a less-permeable layer when the fluid reaches the external drainage radius (r_e) in the most-permeable layer, and r_o is the inner or wellbore radius. Eq. 2 can be used to find r_{pi}.

$$\left(\frac{\phi_i}{k_i}\right) r_{pi}^2 \left[F_r \ln\left(\frac{r_{pi}}{r_o}\right) + \ln\left(\frac{r_e}{r_{pi}}\right) + \frac{(1-F_r)}{2}\right] = \left(\frac{\phi_1}{k_1}\right) r_e^2 \left[F_r \ln\left(\frac{r_e}{r_o}\right) + \frac{(1-F_r)}{2}\right] + r_o^2 \left(\frac{\phi_i}{k_i} - \frac{\phi_1}{k_1}\right)\left[\ln\left(\frac{r_e}{r_o}\right) + \frac{(1-F_r)}{2}\right] \qquad (2)$$

Eqs. 1 and 2 are simplified forms of more general expressions that will be presented later. All of the equations may be derived using mass balances and the Darcy equation. These equations are valid for either constant injection rate or constant pressure drop maintained across the cores.

Eqs. 1 and 2 were used to produce Fig. 1, which compares the degrees of penetration in linear vs. radial parallel floods. The r_e value used in Eq. 2 was 50 ft. [Unless stated otherwise, r_o is 0.5 ft and porosity (ϕ) has the same value in all layers for the remainder of the figures in this paper.] Each curve in Fig. 1 shows how the degree of penetration varies with the permeability contrast between the high- and low-permeability layers (k_1/k_i). Also, for each flow geometry, three resistance factors were examined. Coincidentally, the curve for linear flow with a resistance factor of 100 exactly coincides with the curve for radial flow with a resistance factor of 1.

As expected, Fig. 1 demonstrates that the gelling agent penetrates less into the low-permeability layer as the permeability contrast increases.

For both the linear and the radial flow geometries, Fig. 1 illustrates that the degree of penetration into the less-permeable layer increases with increased resistance factor. This is a basic principle of polymer flooding and has been recognized for many years. In a traditional polymer flood (where improvement of the water-oil mobility ratio is the objective), increased depth of penetration in the low-permeability layer is desirable since it results in an improved vertical sweep efficiency. Thus, in a traditional polymer flood, a high-resistance-factor (high-viscosity or low-mobility) injection fluid is preferred. For gel treatments, in contrast, one normally wants to minimize the degree of penetration in the low-permeability layer because the gel that subsequently forms will hinder oil displacement from that zone. Thus, injection of a low-resistance-factor (low-viscosity) gelling fluid is preferred when placing gels without the benefit of zone isolation methods.

A second important point to be taken from Fig. 1 is that for a given injection fluid and permeability contrast, the degree of penetration into the low-permeability layer is dramatically less in linear floods than in radial floods. This indicates that results from linear laboratory corefloods significantly overestimate the degree of profile modification obtainable in field applications where radial flow occurs.

Flow in Reservoirs. In the analysis to this point, the gelling fluid reached the end of the "reservoir" in the high-permeability layer. This situation is unlikely in actual field practice unless a fracture extends from injection well to production well. Therefore, the analysis must be modified.

A radius or distance of reference, r_{pm}, will be chosen such that r_{pm} is the greatest depth of penetration that the gelling agent will reach in the reservoir. Therefore, the gelling agent will never reach the radius of reference in any layer except that with the highest permeability. If fractures are not present, r_{pm} will be typically in the range of 50 to 100 feet for most near-wellbore treatments. Since most gel treatments are applied after a waterflood has been underway for some time, it is probable that any remaining oil within r_{pm} of the injector is effectively immobile.

A factor, Ψ_i, will also be introduced. For layer i, Ψ_i is defined as the pressure drop between r_{pm} and the production well divided by the pressure drop between the injection well and r_{pm} just prior to the injection of any gelling agents. Thus, Ψ_i is established during injection of water rather than gelling agent. Using the method of Deppe,[13,14] Ψ_i may be approximated for the case of a waterflood in a five-spot pattern using

$$\Psi_i = \frac{\ln\left(\frac{r_{wf}}{r_{pm}}\right) + M \ln\left(\frac{r_e}{r_{wf}}\right) + M \ln\left(\frac{r_e}{r_o}\right)}{\ln\left(\frac{r_{pm}}{r_o}\right)} \qquad (3)$$

if $r_{pm} \leq r_{wf} \leq r_e$ or

$$\Psi_i = \frac{\ln\left(\frac{r_e}{r_{pm}}\right) + \ln\left(\frac{r_e\,(\sqrt{\pi}-1)}{\sqrt{\pi}\,r_e - r_{wf}}\right) + M \ln\left(\frac{\sqrt{\pi}\,r_e - r_{wf}}{r_o}\right)}{\ln\left(\frac{r_{pm}}{r_o}\right)} \qquad (4)$$

if $r_{wf} > r_e$, where $r_e = \sqrt{\frac{A_p}{2\pi}}$

Close examination of Eqs. 3 and 4 indicates that Ψ_i is most strongly influenced by the water-oil mobility ratio (M). As M increases from 1 to 50, Ψ_i increases from 1.9 to 72 (for r_{pm}=50 ft, r_o=0.5 ft, r_{wf}=r_e, 20-acre pattern). The factor Ψ_i is fairly insensitive to well spacing. As the pattern size (A_p) is increased from 5 acres to 80 acres, Ψ_i increases by less than 40%. The value for Ψ_i is also relatively insensitive to the distance between injector and waterfront (r_{wf}) if the position of the waterfront is not too close to either well. Other factors, such as well pattern and areal heterogeneity, can also influence the value of Ψ_i.

With a knowledge of Ψ_i, the injection rate (q_{io}) into zone i just prior to the gel treatment can be expressed as

$$q_{io} = \frac{2\pi k_i h_i \Delta p}{\mu_w\,(1+\Psi_i)\,\ln\left(\frac{r_{pm}}{r_o}\right)} \qquad (5)$$

where Δp is the pressure drop between the injector and the producer. During injection of a Newtonian gelling agent (subsequent to water injection), the instantaneous injection rate (q_i) into zone i is

$$q_i = \frac{2\pi k_i h_i \Delta p}{\mu_w \left[F_r \ln\left(\frac{r_{pi}}{r_o}\right) + \ln\left(\frac{r_{pm}}{r_{pi}}\right) + \Psi_i \ln\left(\frac{r_{pm}}{r_o}\right)\right]} \qquad (6)$$

It is also given by

$$q_i = 2\pi h_i \phi_i r_{pi}\left(\frac{dr_{pi}}{dt}\right) \qquad (7)$$

Eqs. 6 and 7 can be used to derive Eq. 8.

$$\left(\frac{\phi_i}{k_i}\right)\left\{r_{pi}^2\left[F_r \ln\left(\frac{r_{pi}}{r_o}\right) + \ln\left(\frac{r_{pm}}{r_{pi}}\right) + \Psi_i \ln\left(\frac{r_{pm}}{r_o}\right) + \frac{(1-F_r)}{2}\right] - r_o^2\left[(1+\Psi_i)\ln\left(\frac{r_{pm}}{r_o}\right) + \frac{(1-F_r)}{2}\right]\right\} =$$

$$\left(\frac{\phi_1}{k_1}\right)\left\{r_{pm}^2\left[(F_r+\Psi_1)\ln\left(\frac{r_{pm}}{r_o}\right) + \frac{(1-F_r)}{2}\right] - r_o^2\left[(1+\Psi_1)\ln\left(\frac{r_{pm}}{r_o}\right) + \frac{(1-F_r)}{2}\right]\right\} \qquad (8)$$

Eq. 8 may be solved (e.g., using the secant method[15]) to find the radius of penetration into zone i (r_{pi}) when a Newtonian gelling agent has propagated to r_{pm} in the most-permeable zone (layer 1). Note that when Ψ_1=0 and Ψ_i=0, Eq. 8 reduces to Eq. 2. Also, Eq. 8 is applicable for either constant injection pressure or constant injection rate. An assumption made in this derivation is that the position of the waterfront does not move significantly during the process of placing the gel. This assumption should be valid if the waterfront is not in close proximity to either well. (The analysis will be valid if the waterfront has actually reached the production well.)

In the most common application of gel technology in unfractured reservoirs, the most-permeable layer (layer 1) will be watered-out, but the waterfront may be far from the production well in a less-permeable layer (layer i). Since water is effectively the only mobile fluid in layer 1, Ψ_1 will generally be about 2 (which is also the approximate Ψ value if M=1). However, in layer i, Ψ_i could have any value in the practical range from 0.5 (for very light oils) to 50 (for more viscous oils). Fig. 2a and 2b illustrate the penetration behavior while injecting a gelling agent into a reservoir where Ψ_1=2. Four different Ψ_i values were examined, ranging from 0.5 to 50.

Fig. 2a shows that the degree of penetration of gelling agent into the low-permeability layer decreases significantly as Ψ_i increases (if F_r=1). This is true even for relatively low values of permeability contrast between layers. Since large Ψ_i values are associated with high water-oil mobility ratios, Fig. 2a suggests that zone isolation may be needed least when placing gels in reservoirs with relatively viscous oils. Interestingly, gel treatments have been applied rather infrequently in reservoirs with viscous crudes. Fig. 3 shows the results of a survey of 98 field projects that was based on information published in Enhanced Recovery Week, Oil & Gas Journal, Western Oil Reporter and other sources over the past ten years. It reveals that the median oil-water viscosity ratio (at reservoir temperature) was 9--corresponding to M≈2 and Ψ_i≈3 (for unfractured reservoirs with an end-point water-oil permeability ratio of 0.2). Eighty percent of the projects had oil-water viscosity ratios less than 35--corresponding to M≈7 and Ψ_i≈10, and 98% of the projects had oil-water viscosity ratios less than 180--corresponding to M≈16 and Ψ_i≈23.

Increasing the resistance factor of the injection fluid moderates the effect Ψ_i has on the degree of penetration into the low-permeability zone. This is apparent upon comparing Figs. 2a and 2b. Fig. 2b presents results from an analysis identical to that in Fig. 2a except that the resistance factor of the Newtonian gelling agent is 100 rather than 1.

The degree of penetration of gelling agent into less-permeable zones is quite insensitive to r_{pm}. This is illustrated in Fig. 4 for several combinations of Ψ_i and gelling-agent resistance factor (using $\Psi_1=2$, $r_o=0.5$ and $k_1/k_i=100$).

In waterfloods where the waterfront in the most-permeable layer has not reached the vicinity of the production well, Ψ_1 is likely to be similar to Ψ_i values in less-permeable layers. Fig. 5 compares penetration behavior for injection of a fluid ($F_r=10$) into reservoirs having equal Ψ_1 and Ψ_i values. The Ψ_i values range from 0 to 50. (Incidently, the case where $\Psi_1=\Psi_i=0$ is equivalent to that of parallel radial corefloods, and the penetration behavior is identical to that for the radial case where $F_r=10$ in Fig. 1.) Fig. 5 shows that, except at extreme permeability ratios, variations of Ψ_i have a relatively small effect on the penetration behavior. Although not shown, Ψ_i variations have even less effect for injection of a fluid with $F_r=100$, and they have no effect if the injected fluid has the same mobility as that for water.

Summarizing Figs. 2-5, the optimum conditions for minimizing the degree of penetration into productive zones during unrestricted injection of Newtonian gelling agents include:

(1) the gelling agent should have a low resistance factor (high mobility) during placement,

(2) the water-oil mobility ratio should be relatively high,

(3) the most-permeable layer should be watered-out, but the waterfronts should not be close to the production well in the productive zones.

Remember that these conditions apply to unfractured wells where layers are separated by impermeable barriers and zones are not isolated during gel placement. As mentioned earlier, the Ψ_i values will generally be less than 10 in unfractured reservoirs. From Figs. 2-5, it is evident that the degree of penetration of a gelling agent into the less-permeable zone will be 0.01 or greater even for permeability ratios up to 1000:1. More commonly, the permeability contrast between high- and low-permeability layers will be less than 100:1, and the degree of penetration of gelling agent will be 0.1 or more in the less-permeable zones.

Injection of Non-Newtonian Fluids. If a non-Newtonian gelling agent is injected, the resistance factor will vary as the fluid flows away radially from the wellbore. To determine the fluid resistance factor at a given point in the reservoir, the rheology in porous media and the permeability dependence of rheology must be known for the gelling agent. Virtually no data of this type are available for the common polymeric gelling agents, such as xanthan-chromium or polyacrylamide-chromium systems. However, Willhite and Uhl[16] have examined the permeability dependence of rheology for xanthan solutions without crosslinker. By manipulating their relations for the rheology of a 1500-ppm xanthan solution, F_r for an illustrative pseudoplastic fluid can be found

$$F_r = \frac{v^{n-1}\, k_i^{0.512}}{0.679\ \mu_w} \tag{9}$$

where v is fluid flux or superficial fluid velocity, and n is a power-law exponent ($n=0.710\ k_i^{-0.073}$).

For illustrative purposes, a similar equation will be used to describe a shear-thickening fluid

$$F_r = \frac{v^{1-n}\, k_i^{0.512}}{0.679\ \mu_w} \tag{10}$$

The effects of rheology on the penetration behavior (for a well where $\Psi_1=2$ and $\Psi_i=10$) are shown in Fig. 6. In contrast to the cases where Newtonian fluids are used, results for injecting non-Newtonian fluids depend on the total injection rate and/or the injection pressure. The results presented in Fig. 6 were obtained using a constant pressure drop of 500 psi [3.45 MPa] between the injector and the producer. Fig. 6 shows that the Newtonian fluid with $F_r=1$ still penetrates least into the low-permeability zones. It also confirms a previous finding[17,18] that shear-thickening fluids have a greater tendency to penetrate into less-permeable zones. The curves for non-Newtonian fluids in Fig. 6 are illustrative only--the rheology and permeability dependence of rheology for polymeric gelling agents have not been reported to date. Hence, these important properties are currently being determined at the New Mexico Petroleum Recovery Research Center.

Injectivity Loss and Profile Modification After Gelation. Injectivity loss in a well is a common measure used to judge the success of a "profile modification" treatment. Unfortunately, overall injectivity loss is unreliable in this capacity because it does not distinguish between injectivity losses in watered-out zones vs. those in oil-productive zones. Of course, injectivity losses in watered-out zones are beneficial since they reduce channeling of water through the reservoir, while injectivity losses in productive zones are detrimental because they diminish the drive mechanism for displacing oil toward the production well.

Water injectivity into zone i prior to gel placement can be found by rearranging Eq. 5

$$\frac{q_{io}}{\Delta p} = \frac{2\pi k_i h_i}{(1+\Psi_i)\ \mu_w\ \ln\left(\frac{r_{pm}}{r_o}\right)} \tag{11}$$

When water injection is resumed after gel placement, injectivity can be estimated using Eq. 12

$$\frac{q_i}{\Delta p} = \frac{2\pi k_i h_i}{\mu_w\left[F_{rr}\ln\left(\frac{r_{pi}}{r_o}\right) + \ln\left(\frac{r_{pm}}{r_{pi}}\right) + \Psi_i\ln\left(\frac{r_{pm}}{r_o}\right)\right]} \tag{12}$$

where F_{rr} is the residual resistance factor (reduction in the permeability to brine caused by the gel). Eq. 12 assumes that the residual resistance factor in gel-contacted rock is not dependent on distance from the wellbore. (The case where F_{rr} depends on distance will be considered later.) If appropriate, a skin-factor term (which accounts for near-wellbore damage that is not associated with gel) may be incorporated into these equations.

For the case of constant pressure drop, the fraction of the original injectivity in zone i that remains after the gel treatment is

$$\frac{q_i}{q_{io}} = \frac{(\Psi_i+1)\ \ell n\left(\frac{r_{pm}}{r_o}\right)}{\left[F_{rr}\ell n\left(\frac{r_{pi}}{r_o}\right) + \ell n\left(\frac{r_{pm}}{r_{pi}}\right) + \Psi_i \ell n\left(\frac{r_{pm}}{r_o}\right)\right]} \qquad (13)$$

Thus, the injectivity loss in a given zone depends on the degree of penetration of gelling agent, on the permeability reduction (F_{rr}) that results after gelation, and on the Ψ_i value of the zone. Eq. 13 was used to generate Figs. 7a and 7b (Ψ_i=2 for Fig. 7a; Ψ_i=10 for Fig. 7b; r_{pm}=50 ft for both cases).

If zones are not isolated during gel placement, injectivity reductions in the most-permeable zone are invariably accompanied by significant injectivity losses in the less-permeable zones. For example in Fig. 7a, if F_{rr} has a value of 10 in both the high- and the low-permeability layers and if the degree of gel-penetration into the less-permeable zone is 0.01, then injectivity would decline 75% in the most-permeable layer and 31% in the less-permeable layer. This would result in an improved injection profile (a greater fraction of water injected subsequently would enter the less-permeable zone), but the flow capacity in the less-permeable layer would be reduced significantly. Thus, if the most-permeable zone was watered-out and the less-permeable zone was responsible for the oil production, the gel treatment would reduce the producing water-oil ratio, but it would also reduce the oil production rate by 31%. If zones were isolated during gel placement, the producing water-oil ratio could be reduced by a greater amount with no loss of oil-production rate.

The residual resistance factor must fall within a certain range in order for the gel treatment to be effective if zones are not isolated during gel placement. For example in Fig. 7a, F_{rr} must be greater than 3 in order to reduce water flow significantly through the most-permeable zone, but F_{rr} must be less than 100 to prevent injectivity losses in the oil-productive zones from being too severe. Comparison of Figs. 7a and 7b indicates that the desired range for F_{rr} shifts to higher values as Ψ_i increases. Of course, if zone isolation is used during gel placement, very high F_{rr} values would be desirable because the most-permeable layer could be plugged without affecting less-permeable strata.

The residual resistance factor after gel placement in one zone may be quite different from that in other zones. Intuitively, F_{rr} is expected to increase with decreasing permeability (as is the case for uncrosslinked polymers[19]). Then injectivity losses might even be greater in the less-permeable zones than in the most-permeable zone--resulting in a less-favorable injection profile, increased producing water-oil ratio and decreased oil-production rate. This emphasizes the importance of knowing how F_{rr} varies with permeability. Unfortunately, very little of this information is available. Therefore, research is underway in our laboratories to determine the permeability dependence of F_{rr} for several common gel systems.

FRACTURED SYSTEMS

Fractures can exist under a number of different circumstances. For example, fractures may be vertical or horizontal. If a fracture is vertical, the height may extend through part or all of the productive zones. In the following analysis of these cases, the productive strata will be assumed to be separated by impermeable layers.

Vertically Fractured Zones. Since the length of a vertical fracture will generally be large relative to the wellbore radius, flow from the fracture into the rock matrix will be effectively linear. Also, because the "permeability" of a fracture is typically 10^3 to 10^6 times greater than that of the rock matrix, the pressure at any point in the fracture is assumed to be approximately equal to the pressure at the wellbore.[20] Thus, the analysis of fluid penetration into those zones that are cut by the fracture is analogous to that for linear core floods. Given that the fracture has two wings of length, L_f, and each wing has two faces, the injection rate into layer i is

$$q_i = \frac{4\ k_i h_i L_f \Delta p}{\mu_w\ [(F_r-1)\ L_{pi} + (\Psi_i+1)\ L_{pm}]} \qquad (14)$$

where L_{pi} is the distance that the gelling agent has propagated from the fracture face (into the rock matrix) in layer i; and L_{pm} is the maximum distance that the gelling agent will propagate from the fracture face in the most-permeable layer. Here, Ψ has effectively the same meaning as that mentioned earlier--it is the pressure drop between L_{pm} and the production well divided by the pressure drop between the injection well (or the fracture face) and L_{pm} (prior to injection of gelling agents).

The injection rate is also given by

$$q_i = 4\ h_i L_f \phi_i \left(\frac{dL_{pi}}{dt}\right) \qquad (15)$$

Eqs. 14 and 15 can be used to derive Eq. 16, which describes the degree of penetration into layer i when gelling agents have reached L_{pm} in the most-permeable zone (layer 1).

$$\frac{L_{pi}}{L_{pm}} = \frac{\left[(\Psi_i+1)^2 + \left(\frac{\phi_1 k_i}{\phi_i k_1}\right)(F_r-1)(F_r+2\Psi_1+1)\right]^{0.5} - (\Psi_i+1)}{F_r-1} \qquad (16)$$

Eq. 16 reduces to Eq. 1 when Ψ_1=Ψ_i=0.

The Ψ values for vertically fractured zones will generally be higher than those for radial flow. For example, if L_f is substituted for the radius of the injection well and (L_{pm}+L_f) is substituted for r_{pm} in Eqs. 3 and 4, then Ψ is approximately 10 for L_f=50 ft, L_{pm}=50 ft, A_p=20 acres and M=1. The value for Ψ will increase with increased water-oil mobility ratio, fracture length and pattern size.

Fig. 8 shows how the degree of penetration into layers of a vertically fractured system varies with permeability ratio, resistance factor and Ψ value. Although Fig. 8 only illustrates results obtained using $\Psi_1=\Psi_i$, many combinations of Ψ_1, Ψ_i and F_r values have been examined.

Fig. 8 confirms two findings that were mentioned earlier in connection with Fig. 1. First, the degree of penetration of gelling agent in less-permeable zones is less extensive in linear-flow geometries than that in radial-flow geometries. Second, the degree of penetration into less-permeable zones increases with increased resistance factor. A third finding, consistent with that from radial systems, is that the degree of penetration in less-permeable zones decreases with increased Ψ value (remember that Ψ varies roughly in proportion to the water-oil mobility ratio). Note that when injecting a fluid with $F_r=1$, the penetration behavior is independent of the Ψ value (and in fact, is the same for any combination of Ψ_1 and Ψ_i).

For a vertically fractured system, water injectivity in zone i prior to gel placement can be found from

$$\frac{q_{io}}{\Delta p} = \frac{4\, k_i L_f h_i}{\mu_w\, (\Psi_i+1)\, L_{pm}} \tag{17}$$

When water injection is resumed after gel placement, injectivity in zone i can be estimated using

$$\frac{q_i}{\Delta p} = \frac{4\, k_i L_f h_i}{\mu_w\, [(F_{rr}-1)\, L_{pi} + (\Psi_i+1)\, L_{pm}]} \tag{18}$$

For the case of constant Δp, the fraction of the original injectivity in zone i that remains after the gel treatment is

$$\frac{q_i}{q_{io}} = \frac{(\Psi_i+1)}{\left(\frac{L_{pi}}{L_{pm}}\right)(F_{rr}-1) + (\Psi_i+1)} \tag{19}$$

Eq. 19 was used to generate Fig. 9, which shows how injectivity varies with degree of penetration of gelling agent and residual resistance factor for $\Psi_i=10$.

A comparison of Figs. 7b and 9 reveals that gel placement without zone isolation is much more likely to favorably modify injection profiles in vertically fractured systems than in unfractured wells. For example, if the degree of penetration of gelling agent into layer i is 0.01 and $F_{rr}=100$ in layer i and in layer 1, then the injectivity in layer 1 (the most-permeable layer) will be reduced to 10% of the original level for both the unfractured well (Fig. 7b) and the vertically fractured well (Fig. 9). However, the injectivity in layer i will be reduced by 58% in the unfractured reservoir and by only 8% in the vertically fractured well.

The difference in profile modification between fractured and unfractured systems will often be greater than that indicated in the preceding example. Consider a two-layer reservoir in which the permeability of one layer is 100 times that of the other. If $\Psi_1=\Psi_i=10$ and $F_r=1$, then the degree of penetration of gelling agent in the less-permeable zone (from Eqs. 8 and 16) will be 0.09 in an unfractured well and 0.01 in a fractured well. If $F_{rr}=100$, the injectivity in layer 1 (from Eqs. 13 and 19) will be reduced by 90% in both wells. However, in layer i the injectivity losses will be 82% in the unfractured system but only 8% in the fractured system. Thus, gel placement without zone isolation would be far more effective in the fractured well than in the unfractured well.

Unfractured Layers Near Vertically Fractured Zones. A vertical fracture could extend through some zones but not others. Since the "permeability" of a fracture is typically 10^3 to 10^6 times greater than the permeability of a rock matrix, injected fluids will propagate a considerable distance along the length of a fracture while penetrating to a very small extent into the rock matrix. If gel in a fracture (as opposed to gel in the matrix adjacent to the fracture) can effectively restrict flow through the fracture, then gel placement without zone isolation is likely to work well. For example, assume that gel fills a 50-ft-long vertical fracture but extends 0.1 ft radially into the rock matrix of an unfractured zone. If F_{rr} is 100 in both the fracture and the matrix, then the flow capacity of the fracture will be reduced 100-fold, while the injectivity in the unfractured zone will experience only a 25% decrease (from Eq. 13 with $\Psi_i=10$, $r_{pm}=50$ ft, $r_{pi}=0.6$ ft).

On the other hand, if the gel is ineffective in restricting flow in the fracture, then gel placement without zone isolation may be detrimental. For example, assume that gel extends 0.1 ft into rock matrix of a given permeability at all points along a fracture. Also, assume that the gel extends 0.1 ft from the wellbore in an unfractured zone with the same permeability. Furthermore, assume that $F_{rr}=100$ in the rock matrix that contains gel, but $F_{rr}=1$ in the fracture. If $\Psi_i=10$ in both zones, injectivity will still be reduced by 25% in the unfractured zone (again, from Eq. 13 with $r_{pm}=50$ ft, $r_{pi}=0.6$ ft), but injectivity in the fractured zone will be reduced by only 2% (from Eq. 19 with $L_{pm}=50$ ft). These observations emphasize the need to know how effectively gels reduce flow in fractures as well as in rock matrixes.

Horizontal Fractures. Most of the arguments made in the preceding section also apply at least qualitatively to horizontal fractures. Gelling agents will propagate far more quickly in a horizontal fracture than in the rock matrix. If the gel effectively restricts flow in the fracture, then the flow capacity of the fracture can be reduced dramatically without seriously damaging unfractured zones. Otherwise, zone isolation will be needed during gel placement.

OTHER CONSIDERATIONS

Non-Uniform Resistance Factors. The resistance factor and residual resistance factor in a given stratum may vary with distance from the wellbore for a number of reasons. Non-Newtonian behavior was one reason that was mentioned earlier. A second reason is that one component of the gelling formulation (e.g., chromium or aluminum) may be preferentially retained by the rock, so that the gel is crosslinked more strongly near the wellbore. Dilution of the gelling agent through dispersion would have a similar effect. Third, the concentrations of gelling agents are often intentionally varied as the course of a given treatment progresses. In many cases, concentrations are increased toward the end of a treatment--leading to more viscous gelling agents and stronger gels near wellbore. In other processes, alternating slugs of gelling agents are injected that rely on mixing in situ to initiate gelation.

Due to space limitations, a detailed analysis of the effectiveness of the many types of gel treatments can not be presented here. However, one general observation can be made with respect to placement of gels without the use of zone isolation. That is, factors that increase the resistance factor during gel placement will increase the desirability of zone isolation. For example, resistance factors and the degree of penetration into low-permeability zones will increase with increased polymer concentration. Gelation or partial gelation before the placement process is complete can also have this effect.

Chemical Retention. One might argue that since retention of polymers increases with decreasing permeability, retention will help to limit the degree of penetration of gelling agents in low-permeability zones. This increase in polymer retention is accompanied by significant increases in F_r and F_{rr} in low-permeability rock.[19] These effects may readily be taken into account for Newtonian fluids. Chemical retention and inaccessible pore volume may be incorporated by multiplying the right side of Eqs. 7 and 15 by $(1+a_{ri}-a_{vi})$, where a_{ri} is chemical retention for layer i (expressed as volumes of fluid depleted of chemical per pore volume contacted) and a_{vi} is inaccessible pore volume for layer i. To account for variations of resistance factors with permeability, F_r in Eqs. 6 and 14 should be replaced by F_{ri}, where the i subscript refers to layer i. This leads to Eq. 20 as a more general form of Eq. 8

$$\left\{r_{pi}^2\left[F_{ri}\ln\left(\frac{r_{pi}}{r_o}\right)+\ln\left(\frac{r_{pm}}{r_{pi}}\right)+\Psi_i\ln\left(\frac{r_{pm}}{r_o}\right)+\frac{1-F_{ri}}{2}\right]\right.$$

$$\left.-\,r_o^2\left[(\Psi_i+1)\ln\left(\frac{r_{pm}}{r_o}\right)+\frac{1-F_{ri}}{2}\right]\right\}(1+a_{ri}-a_{vi})\left(\frac{\phi_i}{k_i}\right)=$$

$$\left\{r_{pm}^2\left[(F_{r1}+\Psi_1)\ln\left(\frac{r_{pm}}{r_o}\right)+\frac{1-F_{r1}}{2}\right]\right. \tag{20}$$

$$\left.-\,r_o^2\left[(\Psi_1+1)\ln\left(\frac{r_{pm}}{r_o}\right)+\frac{1-F_{r1}}{2}\right]\right\}(1+a_{r1}-a_{v1})\left(\frac{\phi_1}{k_1}\right)$$

and to Eq. 21 as a replacement for Eq. 16.

$$\frac{L_{pi}}{L_{pm}}=\frac{\left[(\Psi_i+1)^2+\left(\frac{\phi_1 k_i}{\phi_i k_1}\right)\left(\frac{1+a_{r1}-a_{v1}}{1+a_{ri}-a_{vi}}\right)(F_{ri}-1)(F_{r_1}+2\Psi_1+1)\right]^{0.5}}{F_{ri}-1} \tag{21}$$

$$-\,\frac{\Psi_i+1}{F_{ri}-1}$$

Table 1 lists values for F_r, F_{rr}, retention and inaccessible pore volume in 12- and 137-md sandstone (from Vela, Peaceman and Sandvik[19]). These values were used in Eq. 20 (with $\Psi_1=\Psi_i=10$) to determine the r_{pi} values listed in Table 2. As expected, higher values for retention and F_r in the less-permeable zones result in a lower degree of penetration. However, the effect is surprisingly small considering the sizes of the differences in retention and F_r. A 13-fold difference in F_r values for the two layers only reduces the degree of penetration in the 12-md layer by about 45% (compare entries 1 and 2 with entry 5). Also, a ten-fold difference in retention values for the layers only reduces the degree of penetration in the 12-md layer by about 40% (compare entries 1, 2 and 5 with entries 3, 4 and 9, respectively).

The injectivity losses that result from injecting polymer into an unfractured well with the above characteristics may be assessed using Eq. 13. The injectivity loss in the 137-md layer is 22% when r_{pm}=50 ft, F_{rr}=2.4, a_{r1}=0.23 and $\Psi_1=\Psi_2$=10. The injectivity loss in the 12-md layer is (1) 9% if F_{rr}=2.4 and a_{r2}=0.23, (2) 8% if F_{rr}=2.4 and a_{r2}=2.37, and (3) 67% if F_{rr}=45 and a_{r2}=2.37. Clearly, the injectivity loss from the higher residual resistance factor outweighs any benefit gained from a lower degree of penetration in the low-permeability zone.

In the above example, the F_{rr} values apply to the case where no crosslinker is present. How much a crosslinker would increase these values is presently unknown. Intuitively, a crosslinker should cause the F_{rr} values in the less-permeable zone to increase at least as much as those in the more-permeable zone. Overall, higher chemical retention will probably not mitigate damage to low-permeability zones during gel placement without zone isolation.

Diffusion and Dispersion. Some researchers[21] have suggested that the small bank of gelling agents that enters a low-permeability layer might experience sufficient dilution by diffusion and dispersion to prevent gelation. To examine this possibility, consider that dispersion in low-permeability strata is likely to be small during gel placement because (1) the gelling agent penetrates a relatively small distance into the reservoir (the size of the mixing zone will be proportional to the square root of the length of the gel bank in a given zone), and (2) the gelling-agent/water mobility ratio is less than or equal to one.

Diffusion can be expected to cause significant dilution of the gelling agent when the quantity $[(Dt_g)^{1/2}/x]$ has a value greater than one, where D is diffusion coefficient, t_g is gelation time, and x is either L_{pi} or $(r_{pi}-r_o)$. For a diffusion coefficient of 10^{-5} cm²/s (typical of low-molecular-weight gelling agents) and a gelation time of one day, $[(Dt_g)^{1/2}/x]$ will be less than one for x values that are greater than 0.03 ft. For a diffusion coefficient of 10^{-8} cm²/s (typical of polymeric gelling agents[22]) and a gelation time of one day, $[(Dt_g)^{1/2}/x]$ will be less than one for x values that are greater than 10^{-3} ft. It was shown earlier that the degree of penetration of gelling agent in unfractured, low-permeability zones will usually be greater than 0.01--corresponding to x=0.5 ft for r_{pm}=50 ft.

Therefore, diffusion and dispersion will probably not significantly reduce damage by gels (particularly gels made from polymeric gelling agents) in low-permeability zones in unfractured wells. Because of the shorter distances involved, diffusion might be more important in laboratory core floods. A more rigorous treatment of the effects of dispersion and diffusion may be made using the methods described by Perkins and Johnston[23] and Crank.[24]

Special Situations. Certain circumstances may be exploited to improve gel placement if zone-isolation methods can not be used. For example, prior to a gel treatment, the productive interval in a well might be plugged with debris, while a watered-out zone is open. The gel treatment could then be applied with a reduced risk of gel entering productive zones. After the gel treatment, acid could be spotted on the productive intervals to remove near-wellbore damage and increase injectivity.

In another case, a watered-out channel may be located some distance above a less-permeable, productive interval. If the tubing between the two zones contains enough buffer fluid (e.g., water) and the permeability contrast is high enough, gelling agents can be placed in the watered-out channel without contacting the productive interval. In the converse case, where a watered-out channel is located below a productive zone, the need for zone isolation during gel placement will be accentuated.

CONCLUSIONS

The following conclusions apply to reservoirs in which crossflow between layers does not occur:

1. Zone isolation is far more likely to be needed during placement of gels in unfractured wells than in fractured wells. Productive zones in unfractured wells may be seriously damaged if zones are not isolated during gel placement.
2. Gel placement without zone isolation should cause the least damage to productive zones in unfractured wells when (a) the gelling formulation exhibits a low resistance factor during placement, (b) the water-oil mobility ratio is relatively high, (c) the most-permeable layer(s) are watered-out, and (d) the waterfronts are not close to the production well in the productive zones.
3. Parallel linear corefloods overestimate the degree of profile modification that can be attained in radial systems (for gel placement without zone isolation).
4. Chemical retention, dispersion and diffusion will probably not significantly mitigate injectivity losses caused by gel penetration into low-permeability zones.
5. A need exists to determine the permeability and velocity dependencies of gelling-agent resistance factors and of gel residual resistance factors.

NOMENCLATURE

A_p = pattern size, acres [m^2]

a_{ri} = chemical retention for layer i expressed as volume of fluid depleted of chemical per pore volume contacted

a_{vi} = inaccessible pore volume for layer i

D = diffusion coefficient, cm^2/s

F_r = resistance factor (brine mobility divided by mobility of the gelling agent)

F_{ri} = resistance factor in layer i

F_{rr} = residual resistance factor (brine mobility prior to gel placement divided by brine mobility after gel placement)

h_i = thickness of layer i, ft [m]

k_1 = permeability of most-permeable layer, md [μm^2]

k_i = permeability of layer i, md [μm^2]

L_f = fracture length, ft [m]

L_{pi} = distance gelling agent has propagated in a linear core or from the face of a vertical fracture (into the rock matrix) in layer i, ft [m]

L_{pm} = maximum distance that gelling agent will propagate from the fracture face in the most-permeable core, ft [m]

L_t = total core length, ft [m]

M = water-oil mobility ratio

n = power-law exponent in Eqs. 9 and 10

Δp = pressure drop between injector and producer, psi [Pa]

q_i = injection rate in layer i, B/D [m^3/s]

r_e = external drainage radius, ft [m]

r_o = wellbore radius, ft [m]

r_{pi} = radius of penetration of gelling agent in layer i, ft [m]

r_{pm} = maximum radius of penetration of gelling agent in most permeable layer, ft [m]

r_{wf} = distance between injector and waterfront, ft [m]

t = time, s

t_g = gelation time, s

v = fluid flux or superficial velocity, ft/d [m/s]

x = r_{pi} or L_{pi}, ft [m]

μ_w = water viscosity, mPa-s

ϕ_1 = porosity associated with the aqueous phase in the most-permeable layer

ϕ_i = porosity associated with the aqueous phase in layer i

Ψ_i = pressure drop between r_{pm} (or L_{pm}) and the production well divided by the pressure drop between the injection well and r_{pm} (or L_{pm}) in layer i prior to gel injection

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SPE 17332

Table 1. Data from Vela, Peaceman and Sandvik[19]

layer	k (md)	F_r	F_{rr}	retention (lb/ac-ft)	a_{ri}*	a_{vi}
1	137	4	2.4	75	0.23	0.32
2	12	51	45	772	2.37	--

* Calculated assuming porosity is 0.2 and polymer concentration is 600 ppm.

Table 2. Effects of Retention (a_r), Inaccessible Pore Volume (a_v) and Resistance Factor (F_r) on Degree of Penetration
Calculations made using Eq. 20 and data from Table 1.
r_{p2} values for 12-md layer when r_{p1}=50 ft in 137-md layer.

entry	F_{r1}	F_{r2}	a_{r1}	a_{r2}	a_{v1}	a_{v2}	$\left(\frac{r_{p2}-r_o}{r_{p1}-r_o}\right)$
1	51	51	0	0	0	0	0.327
2	4	4	0	0	0	0	0.298
3	51	51	0.23	2.37	0	0	0.206
4	4	4	0.23	2.37	0	0	0.178
5	4	51	0	0	0	0	0.172
6	4	51	0	0	0.32	0.32	0.172
7	4	51	0.23	0.23	0	0	0.172
8	4	51	2.37	2.37	0	0	0.172
9	4	51	0.23	2.37	0	0	0.108
10	4	51	0.23	2.37	0.32	0.32	0.098

SPE 17332

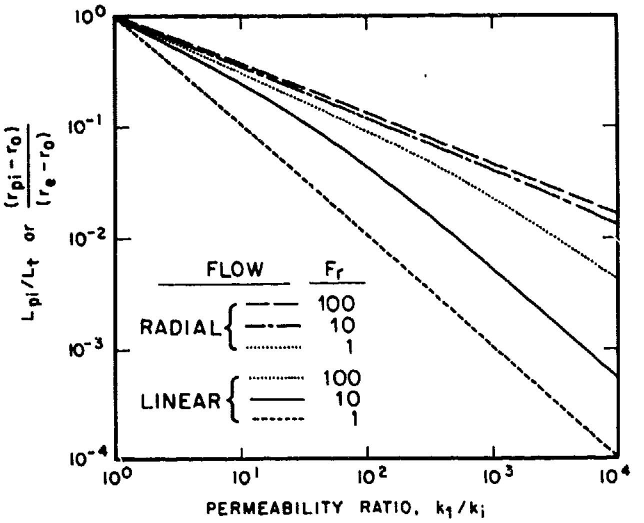

Fig. 1—Radial vs. linear parallel corefloods: degree of penetration into Core i when gelling agent reaches the outlet of the most-permeable core.

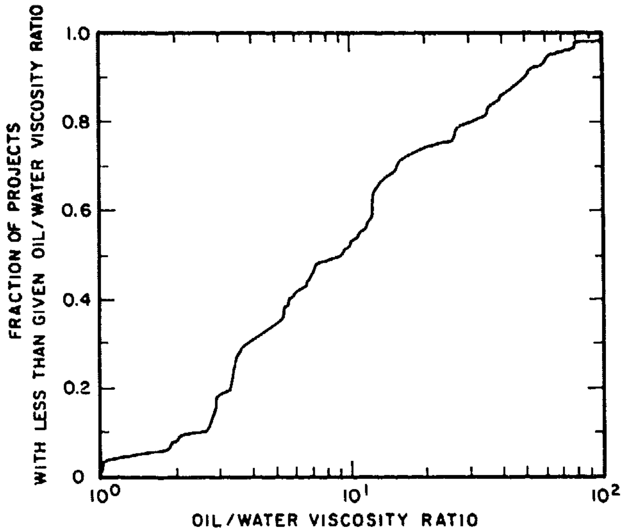

Fig. 3—Oil/water viscosity ratios for 96 profile-modification projects.

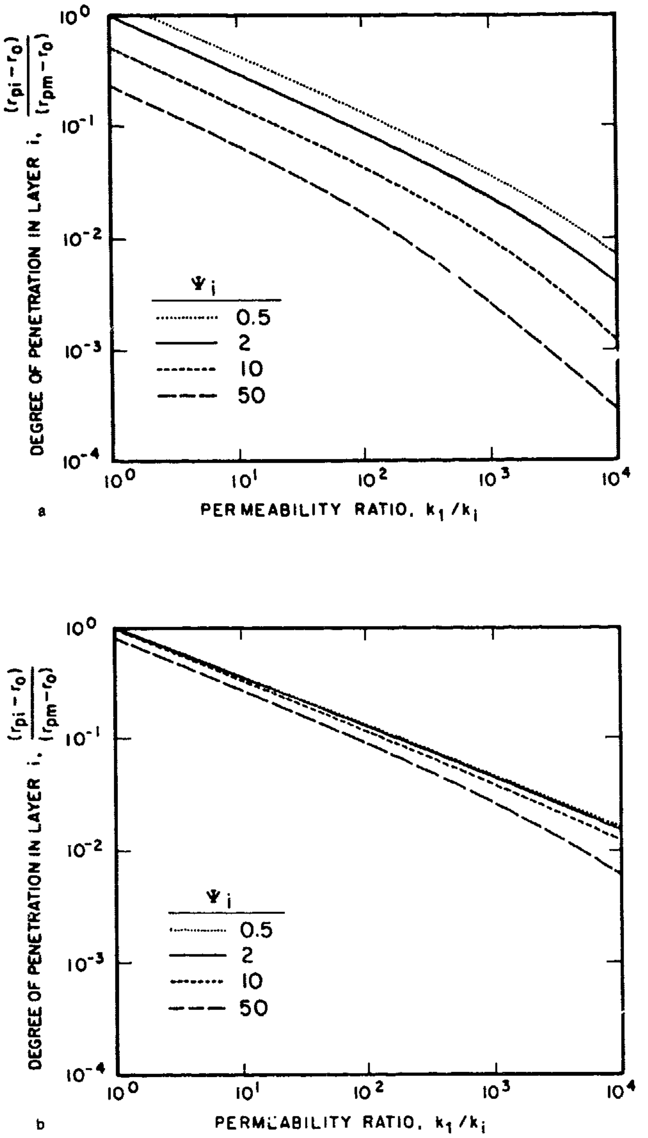

Fig. 2—Effects of permeability ratio, resistance factor, and Ψ on degree of penetration in Layer i when gelling agent reaches r_{pm} in the most permeable layer (Layer 1). Well is not fractured, r_{pm} = 50 ft, r_o = 0.5 ft, Ψ_1 = 2—(a) gelling-agent resistance factor = 1 and (b) gelling-agent resistance factor = 100.

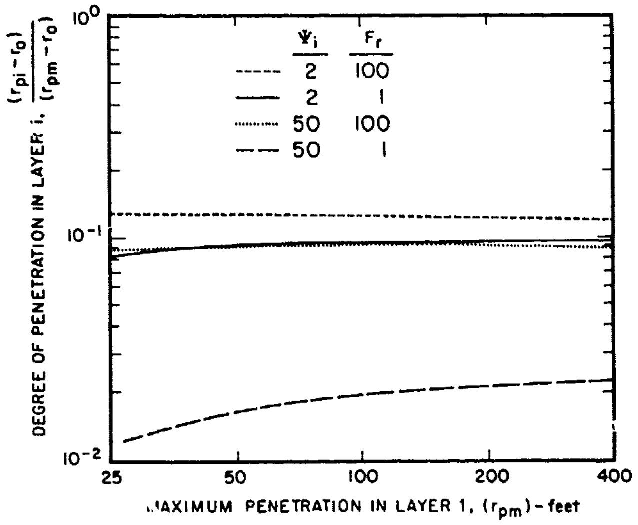

Fig. 4—Effect of r_{pm} on degree of penetration in Layer i when k_1/k_i = 100 and Ψ_1 = 2.

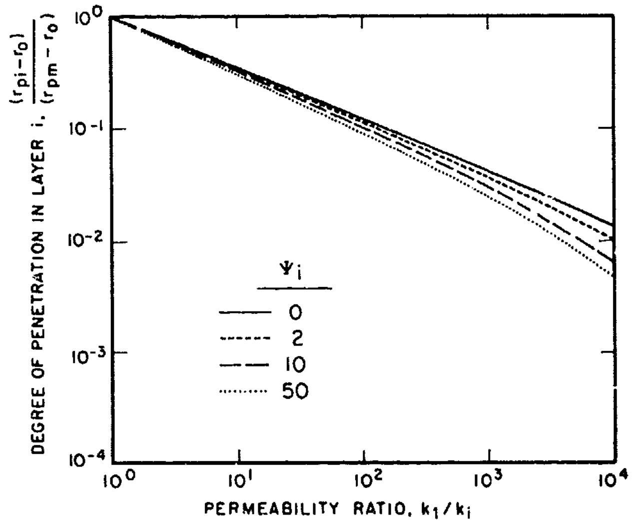

Fig. 5—Effect of Ψ variations when $\Psi_1 = \Psi_i$ and F_r = 10.

SPE 17332

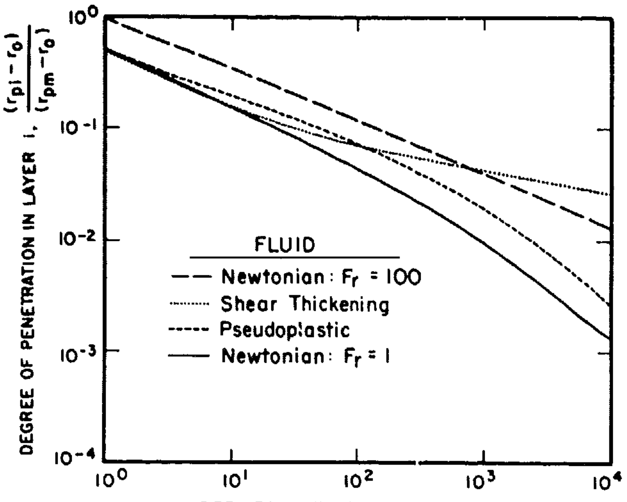

Fig. 6—Effect of rheology on degree of penetration. $\Psi_1 = 2$; $\Psi_i = 10$.

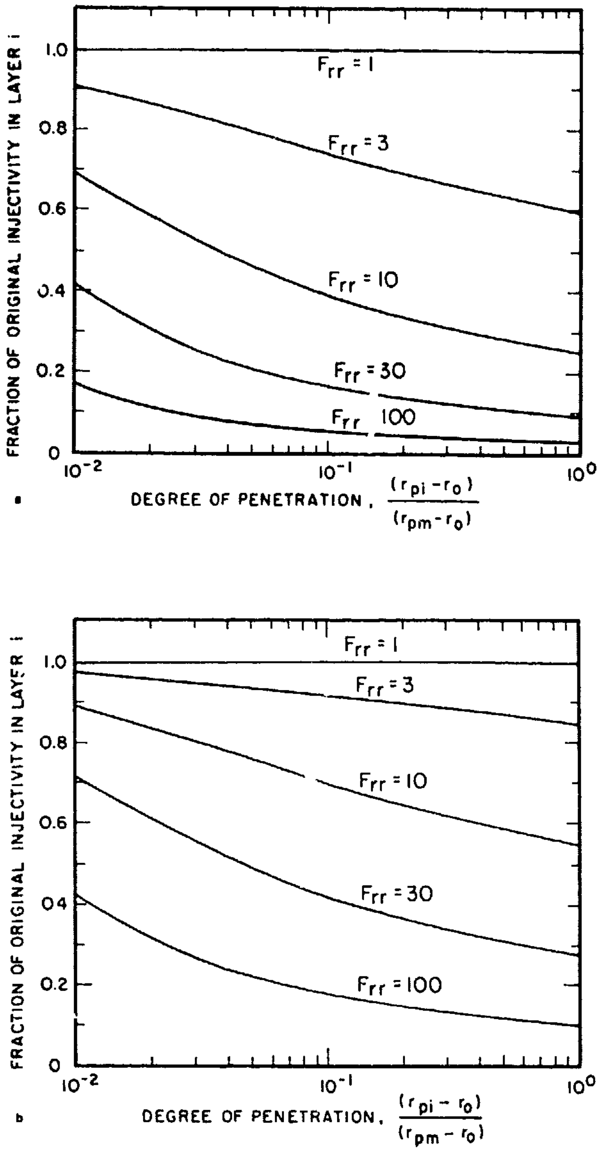

Fig. 7—Effects of degree of penetration, residual resistance factor and Ψ_i on injectivity loss—(a) $\Psi_i = 2$ and (b) $\Psi_i = 10$.

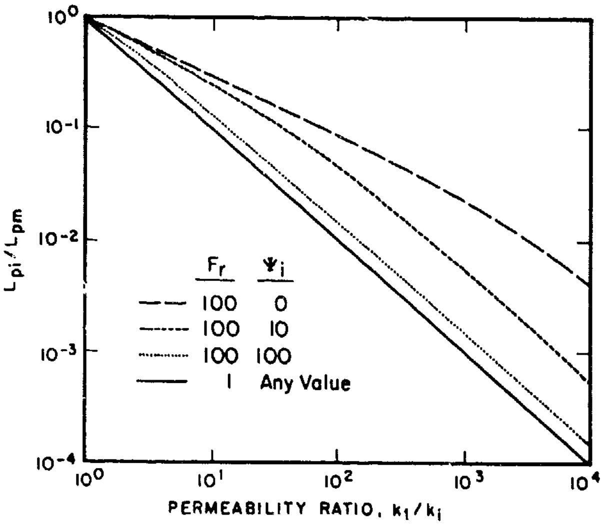

Fig. 8—Effects of permeability ratio, resistance factor, and Ψ on degree of penetration in vertically fractured zones.

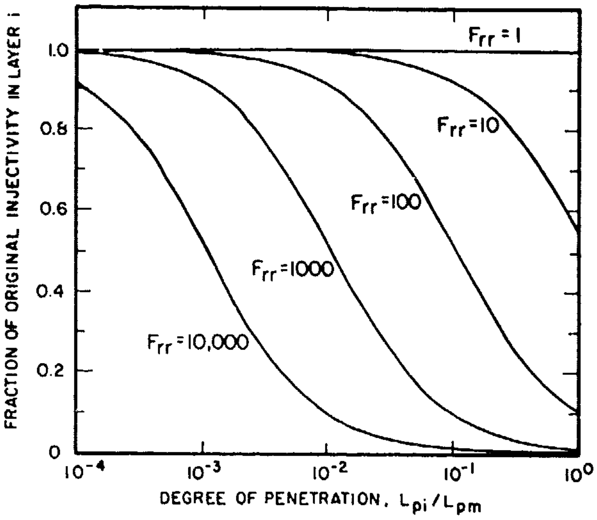

Fig. 9—Effects of degree of penetration and residual resistance factor on injectivity in vertically fractured zones, $\Psi_i = 10$.